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A model for estimation of stress-dependent deformation modulus of rock mass

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ABSTRACT

Deformation modulus of rock mass has a significant role in the support design of an underground excavation. It is determined by expensive in-situ tests or by empirical models. Existing models for estimation of deformation modulus do not consider its stress dependence. Herein, data from several sources is used to develop a stress- (depth-) dependent relation for estimation of deformation modulus. The derived exponential expression incorporates the GSI, Youngs modulus and depth as input parameters for deformation modulus estimate. It is explained that at certain depth, the shear strength of rock joints will become close to the shear strength of a monolithic rock, and below this depth, the rock mass behavior is close to that of a monolithic rock, as well as the deformation modulus.

Keywords : *Deformation modulus, Young's modulus, Rock mass, Stress*

1. Introduction

Designing a support system for underground openings is one of the main tasks in underground mining and construction. The strength of a rock mass is estimated using laboratory tests and geological data with relations provided by rock mass classifications. On the other hand, the rock mass deformability is estimated using empirical relations based on rock mass classification systems or those obtained from in-situ measured data. Measuring the deformation modulus of a rock mass is expensive and time consuming that are the main reasons why the engineering community is trying to establish reliable expressions that will provide the results based on much simpler tests and geological investigations. In-situ measuring assumes that an underground opening exists and it is possible to conducts an experiment at the location of interest. However, measuring the deformation modulus is not possible for deep rock masses where direct access and in-situ testing are not viable. In these cases, the deformation modulus has to be estimated using the existing methodologies.

Many of existing models for estimation of the deformation modulus are obtained by curve fitting to the data obtained from civil tunnels. Those are based on different rock mass classification systems with or without consideration of Young's modulus of the monolith rock.

Table 1 summarizes some of commonly used equations for deformation modulus estimation.

Table 1. Common equations used for deformation modulus estimation.

No.	Equation	Reference
1	$E_m = 2RMR - 100 \quad for \quad RMR > 50$	[1]
2	$E_m = 10^{\frac{RMR-10}{40}} \quad for \quad RMR < 50$	[2]
3	$E_m = 25 log_{10} Q \quad for \quad Q > 1$	[3]
4	$E_m = 0.1(\frac{RMR}{10})^3$	[4]
5	$E_m = \sqrt{\frac{\sigma_c^i}{100}} \cdot 10^{(\frac{GSI-10}{40})}$	[5]
6	$E_m = E_i \left(0.0028 \cdot RMR^2 + 0.9e^{(\frac{RMR}{22.921})}\right)$	[6]
7	$E_m = E_i \cdot 10^{0.0186RQD-1.91}$	[7]
8	$E_m = E_i \cdot \left(0.02 + \frac{1-\frac{D}{2}}{1+e^{\frac{60+15D-GSI}{11}}}\right)$	[8]
9	$E_m = -7.192 + 0.06469 \cdot \sigma_c + 0.20418 RQD + {} + 0.30974 JS + 0.38384 JC + 0.1716 GW$	[9]
10	$E_m = E_i \left(0.5 \cdot \left(1 - \left(\cos \pi \frac{RMR}{100}\right)\right)\right)$	[10]
11	$E_m = 0.5 \cdot MR \cdot \sigma_{ci}$	[11]
12	$E_m = 10 \cdot Q_c^{1/3}$	[12]
13	$E_m = E_i \cdot e^{(RMR-100)/36}$	[13]
14	$E_m = 10 \cdot (RMR - 20)/38$	[14]
15	$E_m = 5.6 \cdot (RMR)^{0.375}$	[15]
16	$E_m = 0.0736 \cdot e^{0.0755RMR}$	[16]

In general, equations presented in Table 1 can be roughly divided into two groups, those that are based on rock mass description only (RMR, Q, GSI) and those that reduce the value of Young's modulus of the monolith rock. Beside these, several significant works have been published by Fattahi and Moradi (2018), Kayabasi et al. (2003), Sonmez et al. (2004), Ajalloeian and Mohammadi (2014), Rezaei et al. (2016) and Beiki et al. (2010) [17-22].

Each of existing models has its own limitations, advantages and disadvantages and care has to be taken when used. The expression given by Hoek et al. (1997) [5], for instance, sometimes overestimates the

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deformation modulus of a rock mass in such manner that the obtained value is larger than Young's modulus of the monolith rock. This was later improved by the work that is based on Young's modulus reduction that provides much realistic values [8].

In addition, there are several models based on the RQD value. This parameter describes the wholeness of the rock core obtained by drilling and not the jointing of the rock mass. On the reliability of RQD, the reader is referenced to the papers by Laubscher and Jakubec (2001) and Jakubec (2007) [23, 24]. Having this considered, expressions based on RQD provide least reliable results.

The stress field in Earth's crust is variable and it is well known that the stress and deformation moduli are mutually dependent [25-27]. The exiting models disregard the stress influence on the deformation modulus and provide the results that are identical for each depth of excavation. Some of those models are based on the data provided by numerous sources where the measuring process was carried out on different depths, but the depth has not been considered in them yet. Herein, the stress-dependent behavior of the deformation modulus is incorporated in the expression that encompasses Young's modulus of a monolith rock, the geological description of a rock mass given by the GSI classification. The expression is derived from curve fitting with measured data given by several sources.

2. Variability of rock mass deformation modulus

Elastic or Young's modulus of rocks is not constant for the whole loading range in unconfined conditions (ex. UCS test). In confined conditions (triaxial test) it is evident that Young's modulus value is changed by different lateral loads. This change is small for very hard rocks and significant for low strength rocks (Fig. 1). When the strength of a rock mass, in the scale of an underground opening, is close to the strength of a small sample of a clastic rock, the deformation curves of the small clastic rock sample (Fig. 1) can be considered as a representative for the rock mass.

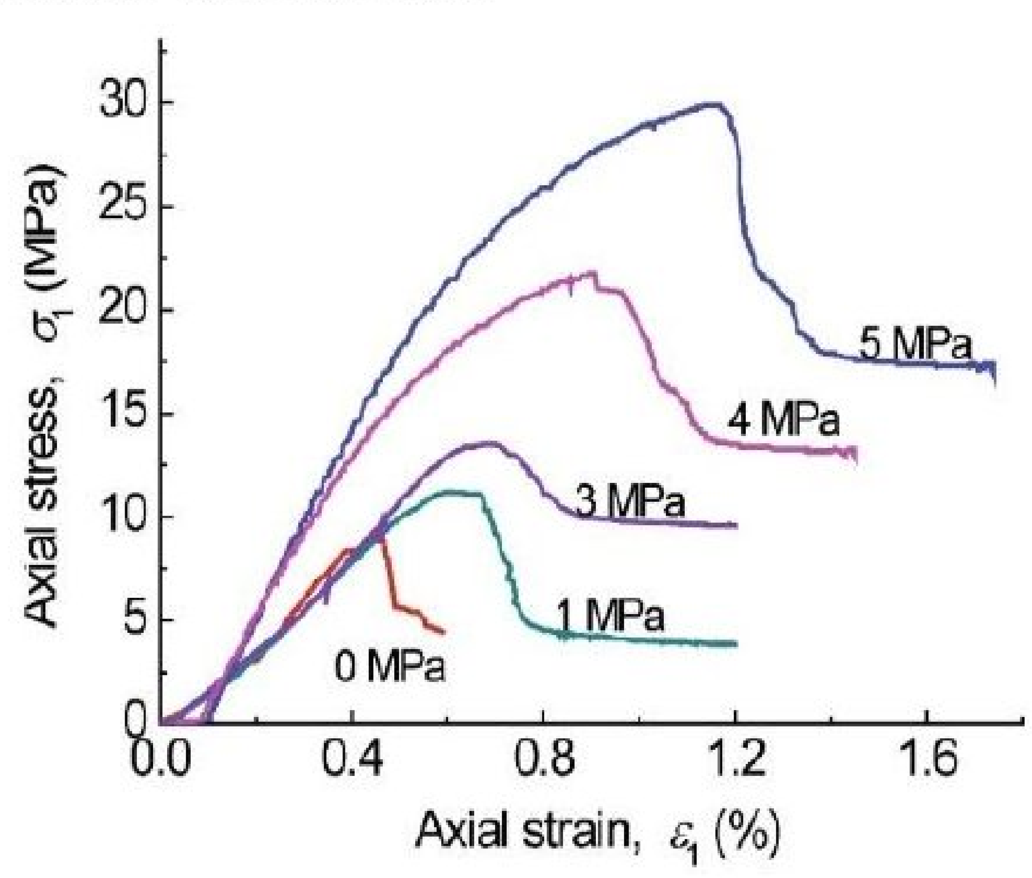

Fig. 1. Young's modulus changes for weak rocks (mudstone) with change in lateral [26].

Kulhawy (1975) proposed the expression about the influence of the lateral stress on the elastic modulus based on a series of investigations [27]:

$$E_r = E_0 \cdot \sigma_3^{\alpha} \tag{1}$$

Where:

E_r - elastic modulus of rock,

E_0 - Youngs modulus determined by Unconfined compression,

σ_3 - lateral stress,

α - factor that has significant value for weak and minimal value for very hard rocks.

Factor a has an average value of 0.14, and only some sedimentary rocks have a value greater than 0.36.

Verman (1997) [28] confirmed the relation by Kulhawy using the reverse analysis based on the measured data from several tunnels. Hereon, the equation where the lateral stress is expressed as depth (H) is obtained:

$$E_m = 0.3H^{\alpha} \cdot E_d \tag{2}$$

Where:

E_d - rock mass deformation modulus,

H - depth,

α - has value in range between 0.16 0.3 for depths below 50m.

In this case, Verman used expression by Bienawski (1973) [29] to determine the rock mass deformation modulus:

$$E_d = 10^{\frac{RMR-20}{38}}$$

This expression does not consider rocks elastic moduli. In cases the assessed RMR was in range 31-68, which is logical, it is indicated that the factor α can be lower and higher for hard and weak rocks. Furthermore, Verman did not propose how to estimate the α values for other RMR values, but only for RMR=31 and RMR=68.

3. Suggestion of the model for rock mass deformation modulus estimation

The suggested model comes from the assumption that the rock mass deformation modulus depends on the deformation of the monolith rock and the deformation of discontinuities. Continuum equivalent of rock mass deformation modulus by Kulhawy (1978) is [30]:

$$\frac{1}{E_m} = \frac{1}{E_r} + \frac{1}{K_n \bar{x}} \tag{3}$$

Where:

E_m - rock mass deformation modulus

E_r - elastic modulus of monolith rock

$\bar{x}$ - average joint spacing

K_n - normal stiffness, after Goodman (1989) [31]:

$$K_n = \frac{d\sigma_n'}{du_n}$$

Where

σ_n' - effective normal stress,

u_n - normal displacement of joint walls

To develop the model, the measured data of rock mass deformation modulus and GSI values are used as presented by Verman (1997) and Cai (2004) [28, 32]. The measured data corresponding the depth of measurement location is presented in Fig. 2.

In these works, Young's modulus of monolith rock is not reported and herein it is estimated using the reversed method suggested by Hoek and Diederichs (2006) [8].

The measured value of rock mass deformation modulus is then separated in part that originates from the monolith rock and the part that originates from discontinuities. The part that originates from the monolith rock is expressed as $E_i \frac{GSI}{600}$. According to the findings of Kulhawy (1975) the elastic modulus of monolith rock changes with lateral stress, or with the gravitational stress component, and therefore the part that originates from the monolith is:

$$E_m^m = \frac{E_i \cdot GSI}{600} \cdot (27z)^{\alpha} \tag{4}$$

Where:

E_m^m - part of def. modulus that originates from the elastic modulus (GPa)

E_i - Young's modulus of monolith rock (GPa)

GSI - Geological strength Index

z - depth (km)

α - coefficient given by Kulhawy (1975) (0.1 used for curve fitting)

The remaining part of the measured value originates from the discontinuities and this is described with the modified GSI value. This modification is carried out in a way that the gravitational stress component is added to the GSI value, or:

$$GSI_m = GSI + 27z \tag{5}$$

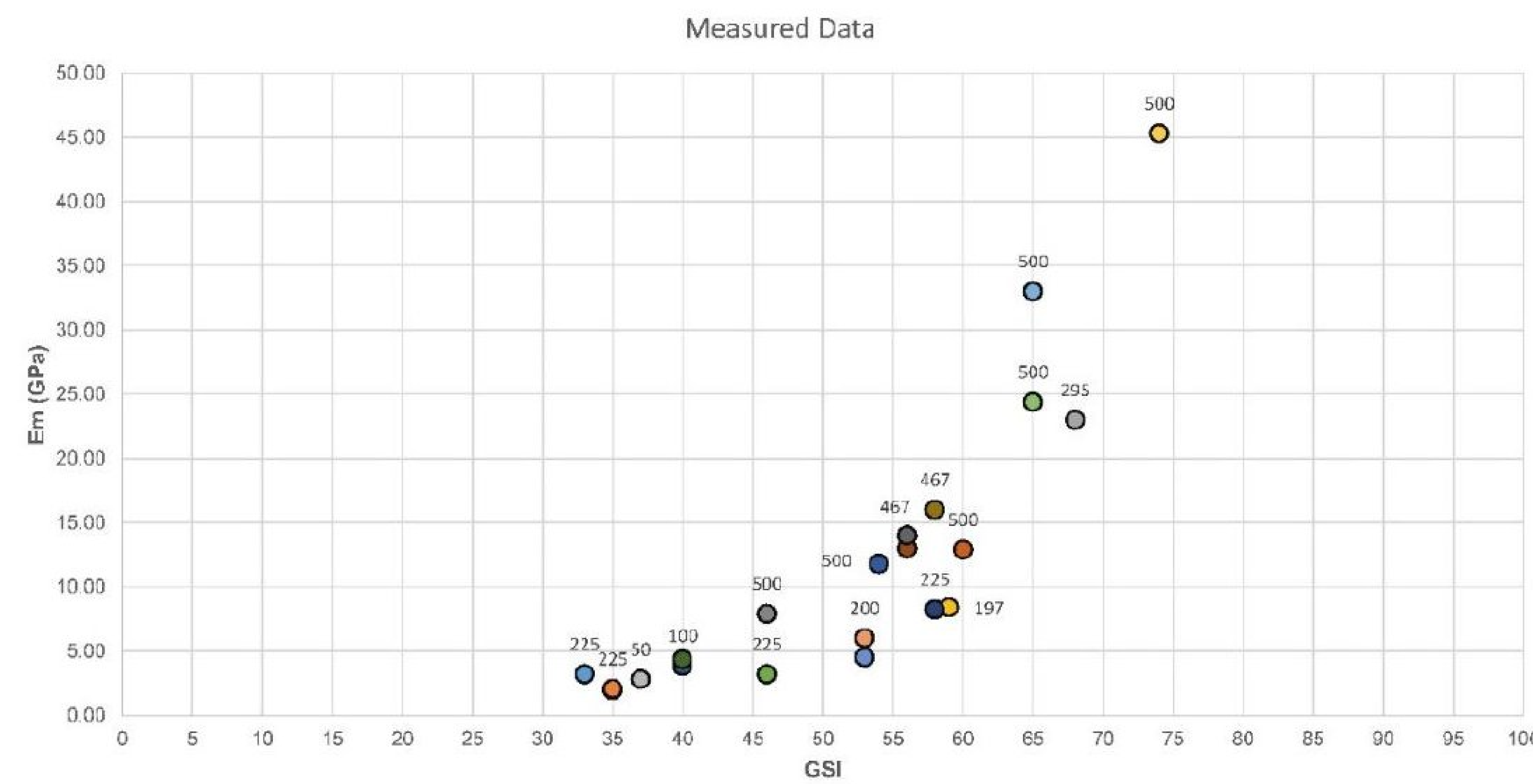

Fig. 2. Rock mass deformation modulus as a function of GSI with corresponding depths [28, 32].

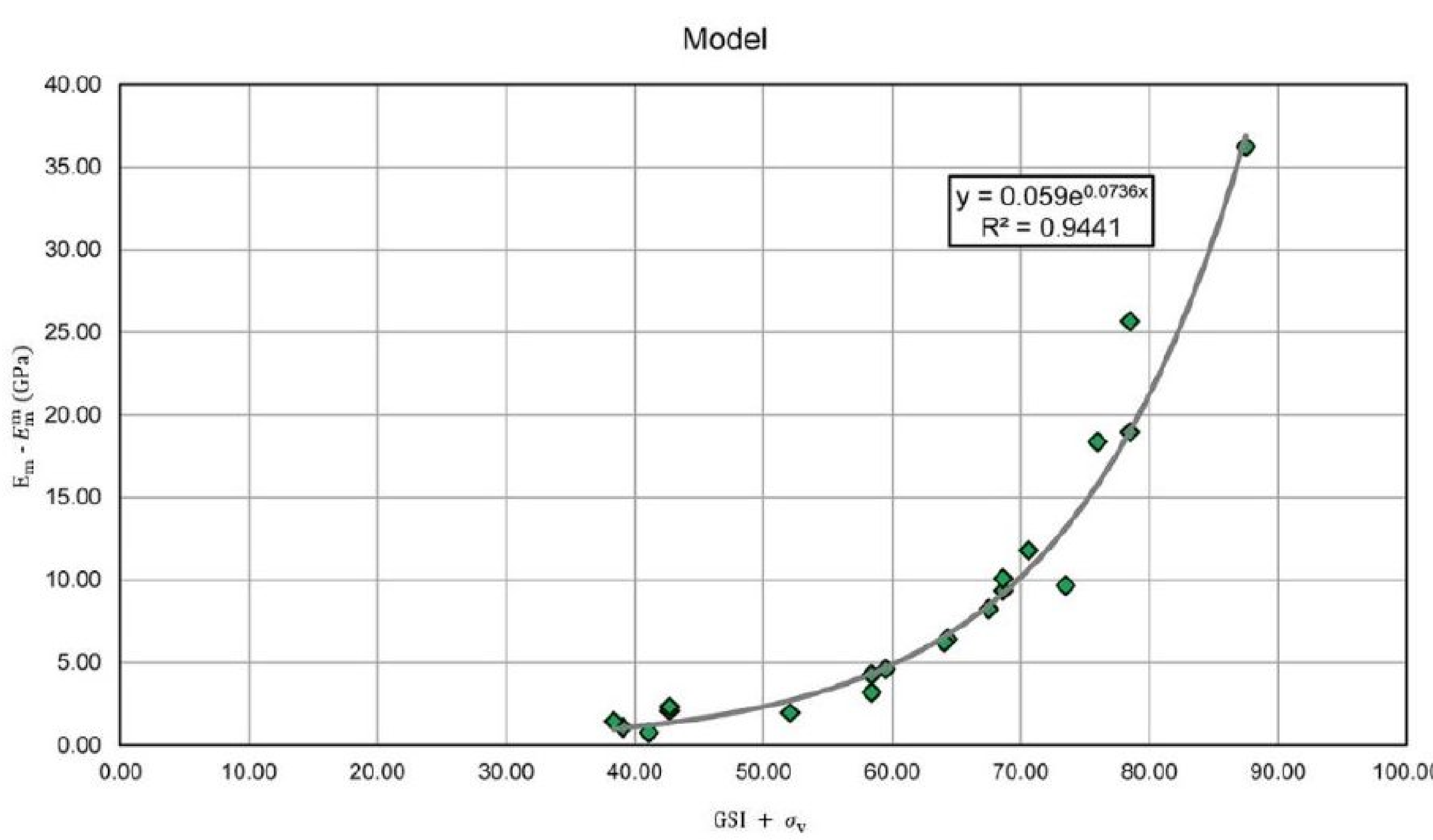

Fig. 3 Exponential curve fit of the model data.

After plotting the points obtained by expressing the part of the measured rock mass deformation modulus value that originates from the discontinuities as a function of modified GSI value, the best fit curve (Fig. 3) through these points is:

$$Y = 0.059e^{0.0736x} \tag{6}$$

The determination coefficient $R^2 = 0.9441$ for the fitted curve.

And therefore, part of the deformation modulus that originates from the discontinuities is:

$$E_m^r = 0.059e^{0.0736(GSI+27z)} \tag{7}$$

And finally, the expression of estimating the rock mass deformation modulus is:

$$E_m = E_m^m + E_m^r \tag{8}$$

or:

$$E_m = 0.059e^{0.0736(GSI+27z)} + \frac{E_i \cdot GSI}{600} \cdot (27z)^{0.1} \tag{9}$$

The measured and calculated values are presented in Table 1.

Rock mass deformation modulus increases with the depth and at certain point, due to the increase of normal force that acts on joint walls, shear strength of discontinuities becomes close to the shear strength of the monolith rock. After this point, the rock mass behaves as the monolith rock and its deformation modulus is equal to Young's modulus of monolith. The depth at which this occurs is herein called *depth limit*. Below this depth, the deformation modulus is increased by an expression given by Kulhawy (1975):

$$E_m = E_i \cdot \left(27z_g\right)^{0.1} \tag{10}$$

Where:

z_g - depth growth (difference between actual depth and depth at which $E_m = E_i$)

Fig. 4 illustrates the change of the rock mass deformation modulus with the depth.

4. Conclusions

The rock mass deformation modulus is a crucial parameter for designing the support system of underground excavations. The amount of required support and its reliability are directly related to this parameter. In order to avoid expensive in-situ tests, it is necessary to estimate it using some of existing expressions. As it was previously discussed, the exiting models do not consider the stress influence on the deformation modulus and the results provided are independent of the excavation depth. However, the dependence of deformation modulus on the stress is well known and herein this is incorporated into the model that is based on geological description of the rock mass (GSI), Young's modulus of monolith rock, and depth.

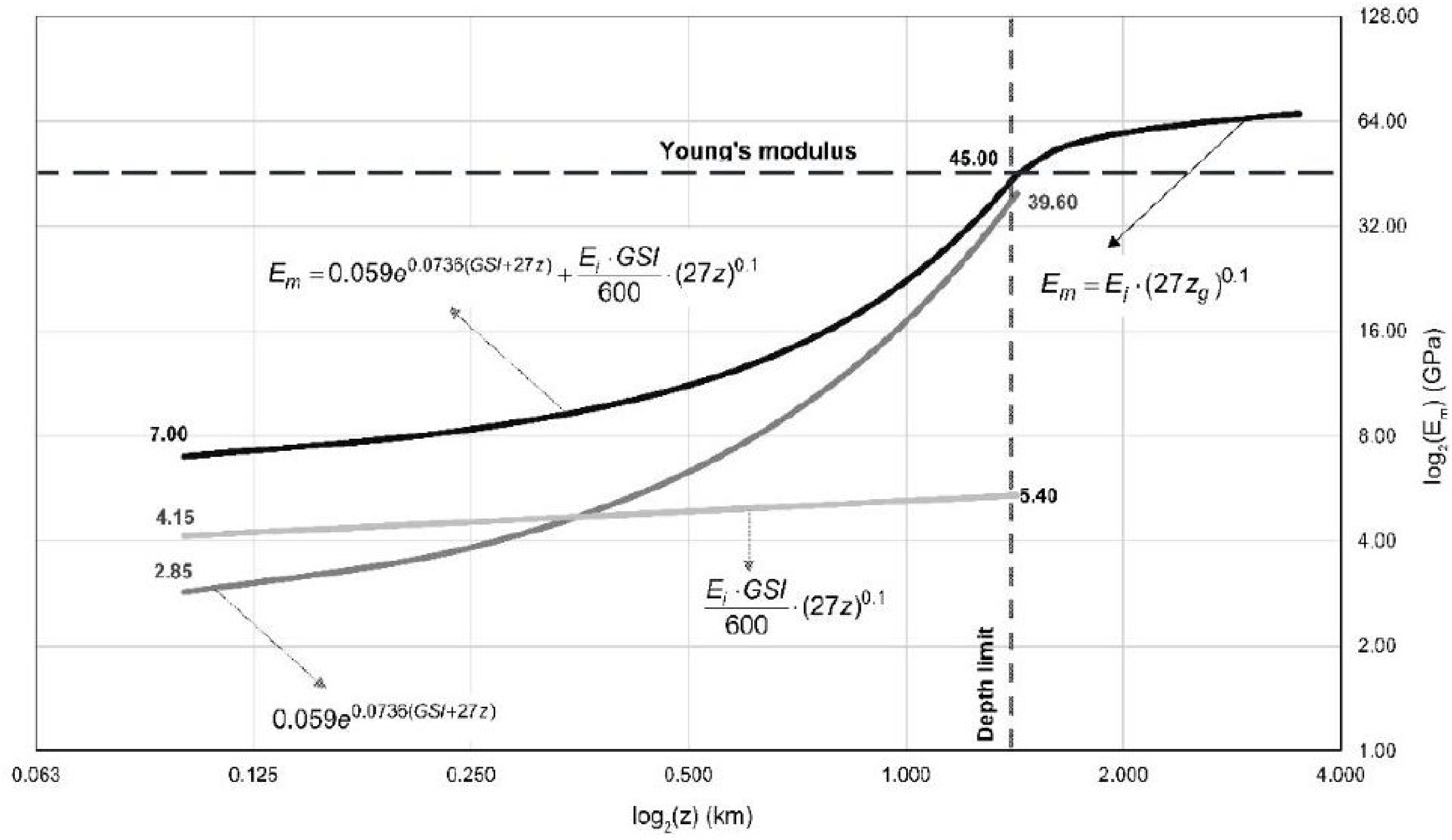

Fig. 4. Rock mass deformation modulus model explanation for GSI = 50 and Ei = 45GPa.

Table 2. Measured and calculated data for the model.

GSI	E_m (GPa)	E_i (GPa)	z (km)	E_m^m	$E_m - E_m^m$	27z (MPa)	GSI_m	Source
35	1.95	17.19	0.225	1.2	0.75	6.08	41.08	[28]
35	2.05	18.08	0.225	1.26	0.79	6.08	41.08	[28]
68	23	33.13	0.295	4.62	18.38	7.97	75.97	[28]
59	8.4	16.89	0.197	1.96	6.44	5.32	64.32	[28]
33	3.2	32.29	0.225	2.13	1.07	6.08	39.08	[28]
46	3.2	13.4	0.225	1.23	1.97	6.08	52.08	[28]
58	8.24	17.36	0.225	2.01	6.23	6.08	64.08	[28]
56	13	30.23	0.467	3.63	9.37	12.61	68.61	[28]
56	14	32.55	0.467	3.91	10.09	12.61	68.61	[28]
58	16	33.71	0.467	4.2	11.8	12.61	70.61	[28]
40	3.85	24.11	0.1	1.78	2.07	2.7	42.7	[28]
40	4.35	27.25	0.1	2.01	2.34	2.7	42.7	[28]
53	4.5	12.29	0.2	1.29	3.21	5.4	58.4	[28]
53	6	16.39	0.2	1.71	4.29	5.4	58.4	[28]
37	2.8	21.54	0.05	1.37	1.43	1.35	38.35	[28]
74	45.3	56.54	0.5	9.05	36.25	13.5	87.5	[32]
65	33	52.24	0.5	7.34	25.66	13.5	78.5	[32]
65	24.4	38.62	0.5	5.43	18.97	13.5	78.5	[32]
54	11.8	30.5	0.5	3.56	8.24	13.5	67.5	[32]
60	12.9	24.81	0.5	3.22	9.68	13.5	73.5	[32]
46	7.9	33.08	0.5	3.29	4.61	13.5	59.5	[32]

The deformation modulus depends on the rock type (Young's modulus) and the properties (shear strength) of joints. The stress increases with depth and is a normal force that acts on the joints. At a certain depth, the shear strength of joints becomes close to the shear strength of the monolith rock, and in this case, the rock mass behavior is close to the behavior of the monolith rock. This means that after a certain depth, the deformation modulus of the rock mass is independently changed the rock joints and depends only on the stress intensity.

The measured data provided by Verman and Cai was used herein for the model constitution. This database contains the structural information of the rock mass (GSI), the measured deformation modulus and the location depth. The measured value of deformation modulus is separated in two parts, one that originates from Young's modulus and one that originates from the rock joints. The part that originates from the rock mass structure is fitted as a function of the modified GSI value. In this manner, an increase in the shear strength of the rock joints is described. At the end, the expression that incorporates GSI, Young's modulus, and depth is obtained for the estimation of rock mass deformation modulus.

The provided expression is limited to a certain depth. The depth at which the deformation modulus becomes close to the Young modulus due to the increased normal force at the rock joints. Below this depth, the deformation modulus changes with depth with the relationship provided by Kulhawy (1975) [27].

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