Assessing Criteria Weights by the Symmetry Point of Criterion (Novel SPC Method) - Application in the Efficiency Evaluation of the Mineral Deposit Multi-Criteria Partitioning Algorithm

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Assessing Criteria Weights by the Symmetry Point of Criterion (Novel SPC Method)–Application in the Efficiency Evaluation of the Mineral Deposit Multi-Criteria Partitioning Algorithm

Zoran Gligorić, Miloš Gligorić*, Igor Miljanović, Suzana Lutovac and Aleksandar Milutinović

Faculty of Mining and Geology, University of Belgrade, Belgrade, 11 000, Serbia

*Corresponding Author: Miloš Gligorić. Email: milos.gligoric@rgf.bg.ac.rs

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ABSTRACT

Information about the relative importance of each criterion or the weights of criteria can have a significant influence on the ultimate rank of alternatives. Accordingly, assessing the weights of criteria is a very important task in solving multi-criteria decision-making problems. Three methods are commonly used for assessing the weights of criteria: objective, subjective, and integrated methods. In this study, an objective approach is proposed to assess the weights of criteria, called SPC method (Symmetry Point of Criterion). This point enriches the criterion so that it is balanced and easy to implement in the process of the evaluation of its influence on decision-making. The SPC methodology is systematically presented and supported by detailed calculations related to an artificial example. To validate the developed method, we used our numerical example and calculated the weights of criteria by CRITIC, Entropy, Standard Deviation and MEREC methods. Comparative analysis between these methods and the SPC method reveals that the developed method is a very reliable objective way to determine the weights of criteria. Additionally, in this study, we proposed the application of SPC method to evaluate the efficiency of the multi-criteria partitioning algorithm. The main idea of the evaluation is based on the following fact: the greater the uniformity of the weights of criteria, the higher the efficiency of the partitioning algorithm. The research demonstrates that the SPC method can be applied to solving different multi-criteria problems.

KEYWORDS

Multi-criteria decision-making; weights of criteria; symmetry point of criterion; mineral deposit; partitioning algorithm; performance evaluation

Nomenclature

MCDM	Multi-criteria decision-making
SPC	Symmetry point of criterion
DM	Decision matrix
ADM	Attribute decision matrix
TMC	Technological mining cut
MCAV	Mining cut attribute vector
BAV	Block attribute vector
EOA	Efficiency of the algorithm

1 Introduction

Multi-criteria decision-making (MCDM) refers to ranking the given set of alternatives with respect to the given set of criteria. Assessing the weights of criteria is a very important phase in most MCDM models. The assessment of the weights of criteria significantly influences the final rank of alternatives. Various methods have been developed to determine the weights of the criteria. The process of the criteria weight determination plays a key role in every multi-criteria decision-making (MCDM) problem. The high impact of the criteria weights is directly reflected on the final ranking i.e., optimal final decision. Generally, methods for criteria weight determination are divided into three groups, such as subjective weighting methods, objective weighting methods and combined weighting methods.

Many authors have used objective weighting methods to obtain the final ranking of alternatives. Krishnan et al. [1] developed a modified procedure of the CRiteria Importance Through Intercriteria Correlation (CRITIC) method, called the Distance Correlation-based CRITIC (D-CRITIC) method. They applied five smartphone models to evaluate the criteria weights by D-CRITIC. The obtained results were compared with four other objective weighting methods and analyzed by three different tests to validate the performance of the D-CRITIC method. Žižović et al. [2] presented a new way for modification of the CRiteria Importance Through Intercriteria Correlation (CRITIC) method, called CRITIC-M. The modification was composed of two approaches. The first referred to the normalization technique of the initial data and the second to the expression for determining the final values of the criteria weights. Cavallaro et al. [3] proposed a fuzzy multi-criteria model for combined heat and power (CHP) systems selection. The developed model was based on a fuzzy Shannon's entropy method to compute the objective weights of the criteria while a fuzzy Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method was applied for the final ranking of the alternatives (CHP technologies). Wang et al. [4] presented a sustainable battery supplier selection for battery swapping stations based on MULTIMOORA method with uncertainty. The uncertainty was expressed as triangular fuzzy numbers describing the behavior of the input data. The weights of criteria were evaluated by the entropy method. They also performed the verification of the proposed methodology through the sensitivity analysis and comparative analysis with other MCDM methods in a case study in Beijing, China. Chodha et al. [5] utilized the TOPSIS technique to select the best industrial robot for arc welding. The eight possible alternatives (robots) with respect to the five criteria were taken for the final decision. The entropy weighting method was used to denote the objective criteria weights. Vavrek et al. [6] applied three objective weighting methods (Coefficient of variation–CV, Standard deviation–SD and Mean weight–MW) integrated with the TOPSIS method for assessing the efficiency of cultural services in the Czech Republic. They represented 26 theatres (alternatives) related to 11 indicators (technical and financial) to evaluate the efficiency of the theatres. Mukhametzyanov [7] presented three objective weighting methods (Entropy, CRITIC and SD) for criteria weight determination and performed a comparative analysis of them. Vujičić et al. [8] suggested a multi–criteria decision–making method for air conditioner selection. They applied two MCDM methods, MOORA and SAW, to select the best alternative, while the criteria weights were determined by two objective weighting methods, Entropy and CRITIC. The numerical example was expressed through the case study to illustrate a comparison analysis of the combined MCDM methods and objective weighting approaches. Yalcin et al. [9] examined several different measures, such as accounting-based performance (ABP), value-based performance (VBP) and overall performance (OP), to evaluate the performance of initial public offering (IPO) firms in the pre-IPO and post-IPO periods. The VIKOR method was utilized for ranking the IPO firms, while objective weighting methods (CRITIC and MW) were applied to determine the weights of criteria. They displayed the results by case study to validate the performance of 16 Turkish IPO firms. Şahin [10] investigated the material selection problem for a flywheel. The grey relational analysis (GRA), TOPSIS and

Organization Rangement Et Synthese De Donnes Relationnelles (ORESTE) method were employed for the final ranking of the materials. The objective weighting methods, SD and CRITIC method, were implemented to establish the significance of criteria. The six approaches based on MCDM methods combined with objective weighting methods were compared and integrated with the Copeland method to choose the best alternative. Wei et al. [11] proposed the extended GRA method for solving the probabilistic uncertain linguistic multiple attribute group decision-making (MAGDM) with unknown attributes (criteria) information. The CRITIC method was used to calculate the objective criteria weights. They were shown a numerical example for site selection of electric vehicle charging stations (EVCS) to verify the developed algorithm. Mešić et al. [12] created a new hybrid model based on a combination of the CRITIC and MARCOS methods for the estimation of the logistics performance index in Western Balkan countries. The weights of six criteria were analysed and determined by the CRITIC method and then the five alternatives (Western Balkan countries) were ranked by the MARCOS method. Keshavarz-Ghorabaee et al. [13] developed a novel method for criteria weight determination, namely MEREC (MEthod based on the Removal Effects of Criteria). They showed and described a detailed computational analysis of the MEREC method step by step. They also presented a comparative analysis with other objective weighting methods for the validation of the introduced method's results through the example. Hadi et al. [14] proposed a combination of the MEREC method and modified TOPSIS method to select the best hospital location for COVID-19 infected patients demonstrated in a real-world case study in Baghdad, Iraq. They showed two main phases of the developed model in which the first phase contains the Internet of Things (IoT) platform represented by geolocation alternatives sites. In contrast, the second phase implements the MCDM techniques represented by a web application system. There are also objective weighting methods that are based on the entropy method, such as Criteria Impact Loss (CILOS) and Integrated Determination of Criteria Weight (IDOCRIW) [15–17].

The subjective weighting methods are widely used to evaluate the importance of criteria in MCDM processes. There are many subjective methods, but only a few will be mentioned in the following literature. Gorcun et al. [18] developed a novel integrated fuzzy model based on fuzzy SWARA and fuzzy CODAS methods for assessing an acceptable road tanker vehicle. They established a set of thirteen criteria and four possible alternatives. Fuzzy SWARA was used to quantify the weights of criteria, while fuzzy CODAS was applied to rank the set of alternatives. Cakar et al. [19] proposed a model for supplier selection in a dairy company based on the fuzzy TOPSIS method. They adopted ten criteria and six supplier cities as a set of alternatives and created an MCDM problem. The weights of criteria were determined by the subjective assessment of the decision-maker using a linguistic approach. The supplier cities (alternatives) were selected by a fuzzy TOPSIS method. Pamučar et al. [20] developed a new subjective weighting method for determining the criteria weights named the Full Consistency Method (FUCOM). Stanujkic et al. [21] proposed a new approach for criteria weights evaluation called PIvot Pairwise RElative Criteria Importance Assessment (PIPRECIA). Keršulienė et al. [22] presented a new subjective approach for criteria weight determination, which is known as step-wise weight assessment ratio analysis method (SWARA). Krylovas et al. [23] introduced a novel KEmeny Median Indicator Ranks Accordance (KEMIRA) method for establishing the attribute preferences and to calculate the criteria weights. Besides, the previous mentioned subjective weighting methods, there are some that should be noted, such as AHP [24], Ranking method [25], SMART [26], WEBIRA [27], OPA [28] and many others.

Besides, the objective and subjective weighting methods, combined (integrated) weighting methods take an important place in the process of assessing criteria weights. Many authors deal with these approaches for determining the criteria weights. Odu [29] presented an overview of the basic weighting methods for estimating criteria weights, including a detailed description of the subjective, objective and combined (integrated) weighting methods. Jahan et al. [30] proposed a framework, including all types of weighting methods for material selection problems. They applied objective and subjective weighting methods to develop a novel combined weighting method for criteria weight evaluation. Chen [31] combined AHP and Entropy into one combined weighting method integrated with the TOPSIS method to create an MCDM model for the building material supplier selection. Ali et al. [32] used an integrated weighting method based on a best-worst method (BWM) as subjective and IDOCRIW as objective weighting methods in the decision-making process for power generation technologies selection. Zavadskas et al. [33] combined objective (entropy, CILOS and IDOCRIW) and subjective (experts' attitudes) weighting methods to obtain an overall (integrated) criteria weight.

Bearing in mind these traditional and well-known MCDM methods that are widely applied in different areas, there are many approaches that provide support to decision-makers by integrating all types of weighting methods with other methods under uncertain environments, such as MARCOS [34], MAIRCA [35], RAFSI [36], MABAC [37], MICMAC [38], TODIM [39], picture fuzzy set [40,41], rough set theory [40], neutrosophic set [42,43], intuitionistic set [44], soft set theory [45] and many other extensions of these methods.

As can be seen from the comprehensive literature review, no authors deal with weighting methods for the purpose of evaluating the efficiency of some partitioning algorithm. Specifically, there is no paper to estimate the efficiency of the mineral deposit partitioning algorithm with the help of weight coefficients. These methods are mostly used for the classical determination of the weights of criteria in different real-life problems within the MCDM framework. Guided by that fact, we tried to develop a completely new approach. The paper presents a novel method for objectively defining the weights of criteria. The objective method of determining the weights of criteria helps decision-makers to reduce the subjectivity in that complex task and to increase the accuracy of the final results. It should be noted that this novel method represents a new approach to estimating and analyzing the efficiency of the multi-dimensional partitioning algorithm. For that purpose, the tendency is to balance all dimensions (criteria) to obtain more efficient results from the partitioning algorithm.

The main aim of this study is to present the methodology of assessing criteria weights based on the characteristic of the criterion that is expressed by its symmetry. The developed method belongs to the class of objective methods. Symmetry Point of Criterion (SPC) is used to calculate the weights of criteria. The symmetry point is located in the middle of the interval [a, b], where a and b present the lower and upper value of the criterion, respectively. Measuring the absolute distance from every criterion value to the symmetry point of the criterion, we create the absolute distance vector. The set of these vectors is presented in the form of a matrix of absolute distances. The modulus for a primal matrix element is defined as the ratio of the averaged absolute distance of the criterion to a value of the criterion element. In this way we form the matrix of moduli of symmetry. Averaging every column of this matrix, we obtain row vector, where every element represents the modulus of symmetry of the given criterion. Finally, the weight of the criterion is calculated by dividing every element of the row vector by the sum of the elements.

The application of the developed method is demonstrated to evaluate the efficiency of the mineral deposit partitioning algorithm. This algorithm presents the base for mine production planning. The main objective of the mineral deposit partitioning algorithm is to divide a deposit into a predefined number of parts, and each part should meet the requirements made up by production planners. Usually, these requirements are presented by the elements of the technological vector. The efficiency of the algorithm can be viewed as a multi-criteria decision-making problem. In that context, part of the mineral deposit can be treated as an alternative, while technological requirements are treated as criteria.

If we take into consideration that each part should have equal values to the technological requirements (technological criteria), it implies the uniformity of the weights of criteria. Optimal partitioning means the equal values of the weights of criteria. In that case, the entropy of the criteria weights vector has the maximum value. Making the comparison between the entropy of the optimal partitioning and real-life partitioning, we can calculate the efficiency of the algorithm.

In this study, after presenting the theory of the proposed method, we illustrate the method through an artificial multi-criteria decision-making problem. A comparative analysis with three objective weights methods (CRITIC, Entropy, Standard Deviation and MEREC) has also been undertaken in Section 3, and the results show that the SPC method is very efficient in assessing the weights of criteria objectively. The applicability of the SPC method in the solving real-life problems is demonstrated in Section 4, where the efficiency of the mineral deposit multi-criteria partitioning algorithm is calculated by using the entropy of the weights of criteria. Section 5 presents the conclusion.

2 The SPC Method

In this section, a new method based on the Symmetry Point of Criterion (SPC) is proposed for assessing the weights of criteria in any multi-criteria decision-making problem. The SPC method uses the symmetry point of criterion, i.e., the modulus of symmetry of the criterion to measure its influence on the weights of criteria. A higher value of the modulus indicates a greater weight of the criterion. The following steps are used to estimate the weights of criteria by objective SPC method.

Step 1: Create the decision matrix

Suppose that there are two finite sets, A and C. Set A considers available alternatives, while set C considers criteria that will be used to assign adequate values to the alternatives. Let m and n denote a total number of alternatives and criteria, respectively. Accordingly, the decision has the following form:

$$DM = \left|x_{ij}\right|_{m\times n} = \begin{vmatrix} A/C & C_1 & C_2 & \dots & C_n \\ A_1 & x_{11} & x_{12} & \dots & x_{1n} \\ A_2 & x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & x_{m1} & x_{m2} & \dots & x_{mn} \end{vmatrix} \quad (1)$$

where:

x_{ij}–an assessment of alternative A_i with respect to a set of criteria

m–total number of alternatives

n–total number of criteria

Step 2: Calculate the Symmetry Point of Criterion (SPC$_j$)

In computing the symmetry point of the criterion only extreme values should be considered. Let $X_{i1} = \{x_{11}, x_{21}, \dots, x_{i1}\}^T; \forall i \in [1, m]$ be a column vector of C_1 criterion values, with respect to a set of alternatives. If the lower and upper values of the interval $[a, b]$ are defined as $a = min\{x_{11}, x_{21}, \dots, x_{i1}\}^T$ and $b = max\{x_{11}, x_{21}, \dots, x_{i1}\}^T$, respectively, then the point c, which is located in the middle of the interval, represents the symmetry point of criterion C_1. The common equation for calculation the symmetry point is as follows:

$$SPC_j = \frac{min\{x_{ij}\} + max\{x_{ij}\}}{2}; i = 1, 2, \dots, m; \forall j \in [1, n] \quad (2)$$

Step 3: Create the matrix of absolute distances

Recall vector X_{i1} and create new extended vector by inserting an SPC_1 point at the specified location. For example, $\boldsymbol{X}_{i1} = \{x_{11}, x_{21}, \ldots, \mathrm{SPC}_1, \ldots, x_{i-1\,1}, x_{i1}\}^T$; $\forall i \in [1, m]$ represents an extended vector. Obviously, some values of the criterion are located on the left side of SPC_1, and some on the right side. Further, the absolute distance from every criterion value to SPC_1 should be calculated: $\{|x_{11} - \mathrm{SPC}_1|, |x_{21} - \mathrm{SPC}_1|, \ldots, |x_{i-1\,1} - \mathrm{SPC}_1|, |x_{i1} - \mathrm{SPC}_1|\}^T$; $\forall i \in [1, m]$. If we take into consideration a primal decision matrix, then the matrix of absolute distances can be represented in the following way:

$$D = \left|\left|d_{ij}\right|\right|_{m\times n} = \begin{vmatrix} |x_{11} - SPC_1| & |x_{12} - SPC_2| & \ldots & |x_{1n} - SPC_n| \\ |x_{21} - SPC_1| & |x_{22} - SPC_2| & \ldots & |x_{2n} - SPC_n| \\ \vdots & \vdots & \ddots & \vdots \\ |x_{m1} - SPC_1| & |x_{m2} - SPC_2| & \ldots & |x_{mn} - SPC_n| \end{vmatrix} \tag{3}$$

Step 4: Create the matrix of the moduli of symmetry

Let $\boldsymbol{D}_{i1} = \{d_{11}, d_{21}, \ldots, d_{i1}\}^T; \forall i \in [1, m]$ be the column vector of the absolute distances concerning criterion C_1. The modulus for a primal matrix element x_{11} is defined as the ratio of the averaged absolute distance of criterion C_1 to a value of element x_{11}: $\frac{\sum_{i=1}^{m} d_{i1}}{m} / x_{11}$. Therefore, the matrix of the moduli of symmetry is of the following form:

$$R = \left|r_{ij}\right|_{m\times n} = \begin{vmatrix} \dfrac{\frac{\sum_{i=1}^{m} d_{i1}}{m}}{x_{11}} & \dfrac{\frac{\sum_{i=1}^{m} d_{i2}}{m}}{x_{12}} & \ldots & \dfrac{\frac{\sum_{i=1}^{m} d_{in}}{m}}{x_{1n}} \\ \dfrac{\frac{\sum_{i=1}^{m} d_{i1}}{m}}{x_{21}} & \dfrac{\frac{\sum_{i=1}^{m} d_{i2}}{m}}{x_{22}} & \ldots & \dfrac{\frac{\sum_{i=1}^{m} d_{in}}{m}}{x_{2n}} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{\frac{\sum_{i=1}^{m} d_{i1}}{m}}{x_{m1}} & \dfrac{\frac{\sum_{i=1}^{m} d_{i2}}{m}}{x_{m2}} & \ldots & \dfrac{\frac{\sum_{i=1}^{m} d_{in}}{m}}{x_{mn}} \end{vmatrix} \tag{4}$$

Step 5: Calculate the modulus of symmetry of criterion

Averaging every column of the previous matrix R, we obtain row vector Q, where every element q represents the modulus of symmetry of the j^{th} criterion. Vector Q is defined as:

$$Q = \left|q_{1j}\right|_{1\times n} = \begin{vmatrix} \dfrac{\sum_{i=1}^{m} r_{i1}}{m} & \dfrac{\sum_{i=1}^{m} r_{i2}}{m} & \ldots & \dfrac{\sum_{i=1}^{m} r_{in}}{m} \end{vmatrix}; \forall j \in [1, n] \tag{5}$$

Step 6: The weights of criteria

Finally, in this step, each objective criterion weight is computed using the vector of moduli of symmetry. The following equations are used for assessing the weights of criteria:

$$W = \left|w_{1j}\right|_{1\times n} = \begin{vmatrix} \dfrac{q_1}{\sum_{j=1}^{n} q_j} & \dfrac{q_2}{\sum_{j=1}^{n} q_j} & \ldots & \dfrac{q_j}{\sum_{j=1}^{n} q_j} \end{vmatrix} \tag{6}$$

Fig. 1 shows the flowchart of the SPC method.

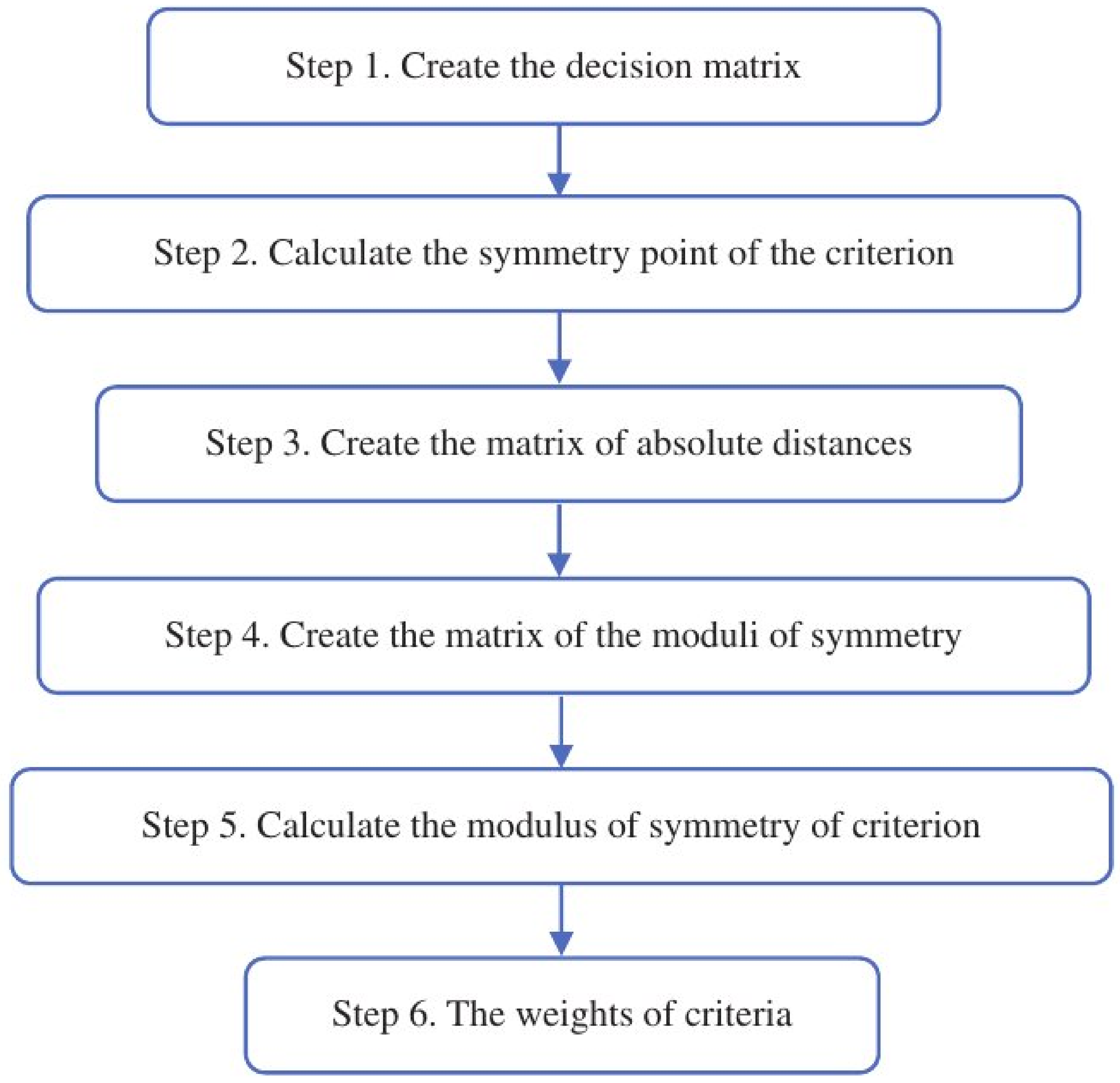

Figure 1: The flowchart of the SPC method

3 The Efficiency of the Mineral Deposit Multi-Criteria Partitioning Algorithm by SPC Method

Underground mining experts are faced with an extremely hard task concerning production planning with respect to the technological requirements. Geostatistical methods use data obtained by exploration drilling to create a block model of a mineral deposit. A mineable block is a three-dimensional object which can be mined in an economically viable way. Each block is characterized by the following attributes: specified dimensions (for example: 10 m × 10 m × 10 m), ore tonnage (for example: 2200 t), ore grades (for example: 1.75% Lead; 3.79% Zinc), and specified *x*, *y,* and *z* coordinates.

The authors borrowed a mineral deposit partitioning algorithm from Gligorić et al. [46] to evaluate its efficiency through the developed SPC method. A hypothetical example borrowed from Gligorić et al. [46] is analysed in the case study. It represents the multi-criteria partitioning algorithm for a mineral deposit. Our idea and motivation of the new proposed method are directed towards the validation of the developed partitioning algorithm. Since the algorithm divided the mineral deposit into several areas that must meet certain technological requirements, we tried to verify these areas by a new approach based on the objective weighting method. The efficiency of our partitioning algorithm is manifested as a uniform distribution of weight coefficients. The new proposed SPC method has proved that the mineral deposit partitioning algorithm succeeded in dividing the mineral deposit to a very high-level, considering the target values of the multi-dimensional *TMCs*.

A mineral deposit can be presented by a finite set of mineable blocks $B = \{b_h\}_{h\in[1,H]}$, where H is the total number of mineable blocks. Characteristics of the blocks are expressed by an attribute vector $BAV_h = \left\{a_j^h\right\}_{j\in[1,A],\forall h\in[1,H]}$, where A is the total number of attributes.

On the other hand, a technological mining cut (TMC) is defined as a subset of B; $TMC_i = \left\{b_l^i\right\}_{l<H;\forall i\in[1,N]}$, where l denotes a total number of blocks associated to the i^{th} TMC, and N denotes a total number of $TMCs$. Obviously, TMC is a union of mineable blocks (the homogeneous part of a mineral deposit) characterized with respect to a given set of technological requirements. The total number

of technological mining cuts equals the number of mining periods (years of mining). Technological requirements are defined by mine planners and can be presented by the vector $C = \{C_j\}_{j\in[1,K]}$, where K denotes the total number of requirements. Each TMC can be described by a mining cut attribute $MCAV_i = \{a_{ic}\}_{c=1,2,..,K;\forall i\in[1,N]}$, where K is the total number of attributes, and equals the number of requirements.

The creation of $TMCs$ can be formulated as a multi-objective optimization problem. To solve this problem, Gligorić et al. [46] developed a multi-criteria clustering algorithm based on the maximization of the similarity between vector $MCAV_i = \{a_{ic}\}_{c=1,2,..,K;\forall i\in[1,N]}$, and $C^{teh} = \{c_j^{teh}\}_{j\in[1,K]}$, where C^{teh} represents the required technological vector. A solution of the formulated problem is given in the following form:

$$B = \cup_{i=1}^{N} TMC_i; TMC_\alpha \cap TMC_\beta = \emptyset; \alpha \neq \beta \tag{7}$$

Accordingly, we can draw an analogy between the MCDM problem and mineral deposit partitioning. Technological mining cuts can be treated as alternatives and require a technological vector as a vector of criteria. This analogy can be presented by the following decision matrix:

$$ADM = \left|z_{ij}\right|_{N\times K} = \begin{vmatrix} A/C & c_1^{teh} & c_2^{teh} & \dots & c_K^{teh} \\ TMC_1 & z_{11} & z_{12} & \dots & z_{1K} \\ TMC_2 & z_{21} & z_{22} & \dots & z_{2K} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ TMC_N & z_{N1} & z_{N2} & \dots & x_{NK} \end{vmatrix} \tag{8}$$

where

z_{ij}–a value of the i^{th} TMC with respect to the j^{th} technological criteria.

The main idea of the efficiency evaluation of the mineral deposit partitioning algorithm is based on the following theorem:

Theorem: for a discrete uniform probability density function p on a finite set of the weights of criteria $(w_1, w_2, \dots, w_j), \forall w_j \geq 0, \sum_{j=1}^{K} w_j = 1$, entropy has the maximum value:

$$H\left(\{W_j\}_{j\in[1,K]} \sim U(w_1, w_2, \dots, w_j)\right) \to max \tag{9}$$

In terms of algorithm efficiency, the theorem can be formulated as: the greater the uniformity of the weights of $\{c_1^{teh}, c_2^{teh}, \dots, c_K^{teh}\}$ criteria, the higher the efficiency of the partitioning algorithm. It means that the weights of criteria should be equal as far as possible.

Proof: The divergence between two discrete distributions p and q is defined as [47]:

$$D(p||q) \triangleq E\left[log\frac{p(W^{teh})}{q(W^{opt})}\right] \tag{10}$$

where

$p(W^{teh})$–discrete distribution of the weights of the technological criteria realized by the SPC method.

$q(W^{opt})$–discrete distribution of the optimal weights of the technological criteria.

Further, U denotes the discrete uniform distribution on $W^{opt} = \{w_1^{opt}, w_2^{opt}, \dots, w_j^{opt}\}, j \in [1, K]$, while p denotes an arbitrary discrete distribution on $W^{teh} = \{w_1^{teh}, w_2^{teh}, \dots, w_j^{teh}\}, j \in [1, K]$. Entropy, for a probability density function p on a finite set $(w_1, w_2, \dots, w_j), \forall w_j \geq 0, \sum_{j=1}^{K} w_j = 1$, is defined by the Shannon equation [48]:

$$H\left(p\left(w_1, w_2, \ldots, w_j\right)\right) = -\sum_{j=1}^{K} p\left(w_1, w_2, \ldots, w_j\right) \cdot log\left(p\left(w_1, w_2, \ldots, w_j\right)\right) \tag{11}$$

The non-negativity of relative entropy implies that:

$$D\left(p||U\right) \geq 0 \tag{12}$$

Hence,

$$D\left(p||U\right) = \sum_{j=1}^{K} p\left(w_j\right) \cdot log\frac{p\left(w_j\right)}{U\left(w_j\right)} = \sum_{j=1}^{K} p\left(w_j\right) \cdot log\left(p\left(w_j\right)\right) - \sum_{j=1}^{K} p\left(w_j\right) \cdot log\left(U\left(w_j\right)\right) \tag{13}$$

$$D\left(p||U\right) = -H\left(p\left(w_1, w_2, \ldots, w_j\right)\right) - \sum_{j=1}^{K} p\left(w_j\right) \cdot log\left(U\left(w_j\right)\right) \tag{14}$$

Note that $U\left(w_j\right) = \frac{1}{K}, \forall j \in [1, K]$. Obviously, $U\left(w_j\right)$ is a constant, and we can pull it out of the summation and rewrite the previous equation as:

$$D\left(p||U\right) = -H\left(p\left(w_1, w_2, \ldots, w_j\right)\right) - log\left(U\left(w_j\right)\right) \sum_{j=1}^{K} p\left(w_j\right) \tag{15}$$

According to the constraint $\sum_{j=1}^{K} p\left(w_j\right) = 1$, we can substitute 1 for expression $\sum_{j=1}^{K} p\left(w_j\right)$. Since 1 is also a probability distribution, the following equality holds:

$$1 = \sum_{j=1}^{K} \left(U\left(w_j\right)\right) \tag{16}$$

This substitution gives us:

$$\begin{aligned} D\left(p||U\right) &= -H\left(p\left(w_1, w_2, \ldots, w_j\right)\right) - log\left(U\left(w_j\right) \sum_{j=1}^{K} \left(U\left(w_j\right)\right)\right) \\ &= -H\left(p\left(w_1, w_2, \ldots, w_j\right)\right) + H\left(U\left(w_1, w_2, \ldots, w_j\right)\right) \end{aligned} \tag{17}$$

where $H\left(U\left(w_1, w_2, \ldots, w_j\right)\right)$ defines the entropy of the uniformly distributed optimal weights of the technological criteria.

According to the above discussion, we have shown that:

$$D\left(p||U\right) = -H\left(p\left(w_1, w_2, \ldots, w_j\right)\right) + H\left(U\left(w_1, w_2, \ldots, w_j\right)\right) \tag{18}$$

Applying the fact that $D\left(p||U\right) \geq 0$ we can conclude that:

$$H\left(U\left(w_1, w_2, \ldots, w_j\right)\right) \geq H\left(p\left(w_1, w_2, \ldots, w_j\right)\right) \tag{19}$$

If we take into consideration that the probability distribution on $W^{teh} = \left\{w_1^{teh}, w_2^{teh}, \ldots, w_j^{teh}\right\}, j \in [1, K]$ was arbitrary, the proof is completed. Thus, it means the theorem: "the greater the uniformity of the weights of criteria, the higher the efficiency of the partitioning algorithm", is valid and can be used to compute the efficiency of the algorithm.

The efficiency evaluation of the algorithm is performed through the following steps:

Step 1: Calculate the weights of criteria by the application of the SPC method over the *ADM* matrix data, and present them through the following vector:

$$W^{teh} = \left\{w_j^{teh}\right\}_{j \in [1,K]} = \left\{w_1^{teh}, w_2^{teh}, \ldots, w_K^{teh}\right\} \tag{20}$$

Step 2: Calculate the optimal values of the weights of criteria as follows:

$$W^{op} = \left\{w_j^{opt}\right\}_{j\in[1,K]} = \left\{w_1^{opt} = w_2^{opt} = \cdots = w_j^{opt} = \frac{1}{K}\right\}_{j\in[1,K]} \tag{21}$$

Step 3: Compute the entropy of the calculated weights. Shannon [48] developed an entropy theory based on thermodynamic that can be interpreted as the quantity of information needed to define the physical state of a system. Entropy can be considered as a measure of the degree of information regularity in weights. The amount of information related to the weights of criteria are calculated using the equations below:

$$H^{teh} = -\sum_{j=1}^{K} w_j^{teh} \cdot log\left(w_j^{teh}\right) \tag{22}$$

$$H^{opt} = -\sum_{j=1}^{K} w_j^{opt} \cdot log\left(w_j^{opt}\right) \tag{23}$$

Step 4: Calculate the efficiency of the algorithm (*EOA*). Discussion about the formulated theorem indicates that entropy of the optimal values of the weights of criteria can be set up as a target value. Based on this, the efficiency of the partitioning algorithm can be calculated as follows:

$$EOA = \frac{H^{teh}}{H^{opt}} \times 100\% \tag{24}$$

where

H^{teh}–entropy of the technological weights of criteria obtained by the SPC method.

H^{opt}–entropy of the optimal weights of criteria (target value).

Transformation of the numerical values of the *EOA* to the linguistic variables is presented in Table 1.

Table 1: Linguistic variables for *EOA*

Value of *EOA* (%)	Linguistic variable
(0–20]	Very low
(20–40]	Low
(40–60]	Moderate
(60–80]	High
(80–100]	Very high

The required technological vector C^{teh} is composed of the following components:

c_1^{teh}–tonnage of a TMC.

c_2^{teh}–compactness (homogeneous) of a TMC, expressed by Schwartzberg's index [49]; it is defined as the ratio of the square perimeter of the shape to the area.

$c_{3,\gamma}^{teh}$–standard deviation of the ore grade in a TMC with respect to the type of a metal hosted in a deposit (%). If $\gamma \geq 2$ then deposit is polymetallic.

A brief description of the data needed to execute the partitioning algorithm is presented in Tables 2, 3, and Fig. 2 (for more details see [46]).

Table 2: Data needed to execute algorithm

Parameter	Description of parameters	Value
Number of mineable blocks	Ore body is divided into blocks with defined dimensions that should belong to one of the technological mining cuts (*TMCs*)	115
Number of technological mining cuts	Ultimate number of mining zones created by certain number of mineable blocks corresponding to planned time for excavated	5
Polymetallic deposit	Typical mineral deposit composed of two related metallic ores	Lead; Zinc
Annual capacity of production (target value)	Total sum of ore tonnage in blocks contained within technological mining cuts (*TMCs*) that should be mined for one year	172 449 t/year
Target value of the *TMC* compactness	Tendency of each created *TMC* to have the approximate shape of a regular square	16 (compactness of the regular square)

Table 3: Tonnage of mineable blocks

Block	Tonnage (t)	Block	Tonnage (t)	Block	Tonnage (t)
1	5891	41	7543	81	8094
2	7604	42	7711	82	10144
3	5646	43	7375	83	7038
4	7849	44	7880	84	8048
5	8461	45	7206	85	7206
6	7727	46	8048	86	7880
7	8583	47	7038	87	7375
8	6350	48	8216	88	7711
9	7497	49	6870	89	7543
10	6120	50	6809	90	7543
11	8186	51	7910	91	7711
12	7635	52	6992	92	7375
13	8323	53	7727	93	7880
14	7497	54	7176	94	7206
15	8461	55	7543	95	8048
16	7359	56	7359	96	9883
17	8461	57	7359	97	6579
18	6564	58	7543	98	7772
19	7635	59	7176	99	6778

(Continued)

Table 3 (continued)

Block	Tonnage (t)	Block	Tonnage (t)	Block	Tonnage (t)
20	6350	60	7727	100	7574
21	7849	61	6992	101	6977
22	6135	62	7910	102	7375
23	7574	63	6809	103	7176
24	7421	64	8094	104	7176
25	8186	65	6625	105	7375
26	7268	66	9974	106	6977
27	8339	67	6809	107	7574
28	7176	68	7910	108	6778
29	7375	69	6992	109	7421
30	6977	70	7727	110	8033
31	7574	71	7543	111	7574
32	6778	72	7359	112	7727
33	7772	73	7359	113	7727
34	6579	74	7543	114	7880
35	7971	75	7176	115	7574
36	7206	76	7727		
37	7880	77	6992		
38	7375	78	7910		
39	7711	79	6809		
40	7543	80	7543		

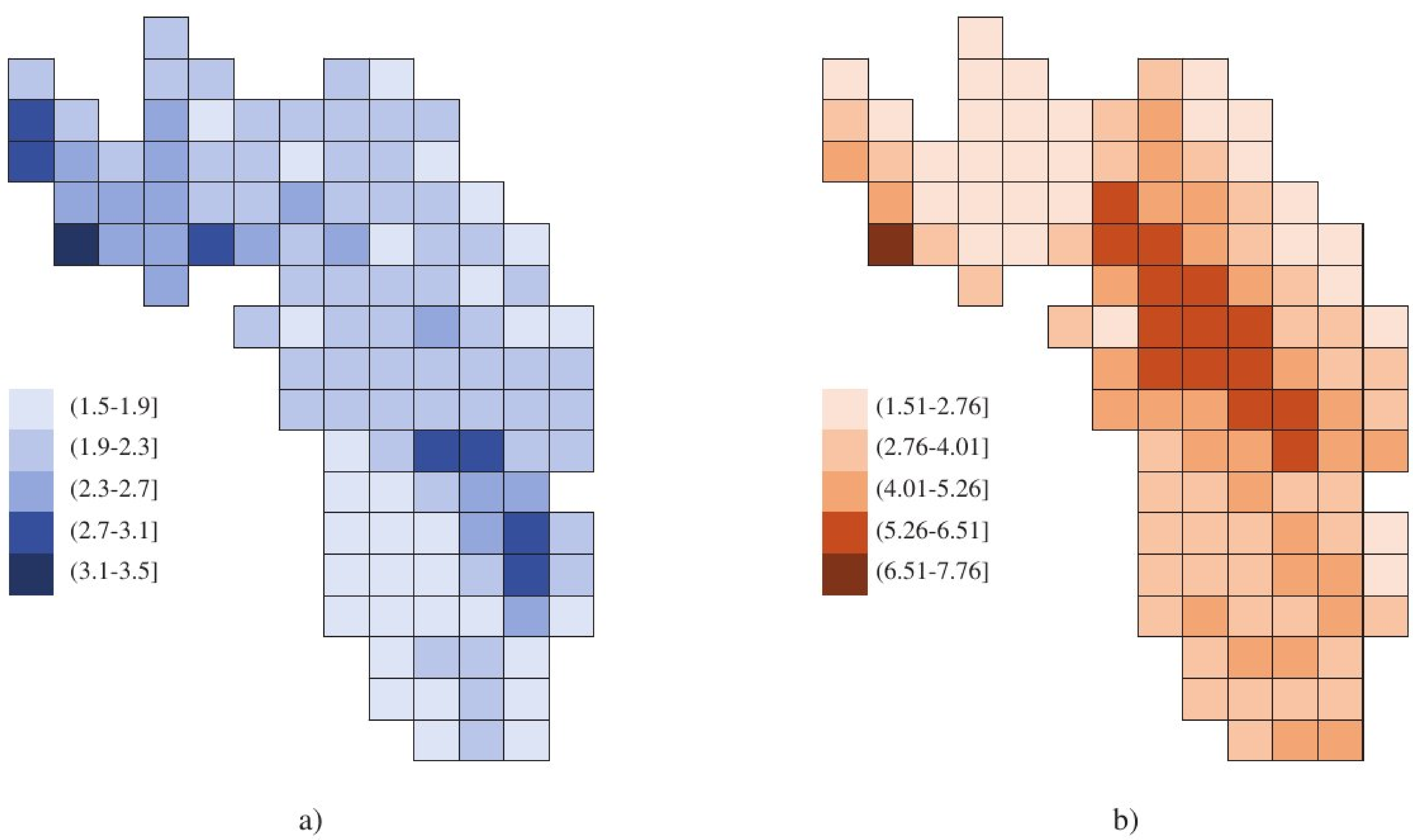

Figure 2: Grade distribution (%): (a) Lead; (b) Zinc [46]

The partitioning algorithm produced five *TMCs* whose characteristics are presented in Table 4, and Fig. 3.

Table 4: Results of the partitioning algorithm

TMC	Tonnage (t) c_1^{teh}	Compactness c_2^{teh}	Standard deviation of the grade-Lead (%) c_3^{teh}	Standard deviation of the grade-Zinc (%) c_4^{teh}
1	169601	39.13	0.325	1.198
2	170241	30.72	0.292	0.418
3	172156	29.39	0.381	0.820
4	172859	62.78	0.169	0.928
5	177388	48.16	0.267	1.262

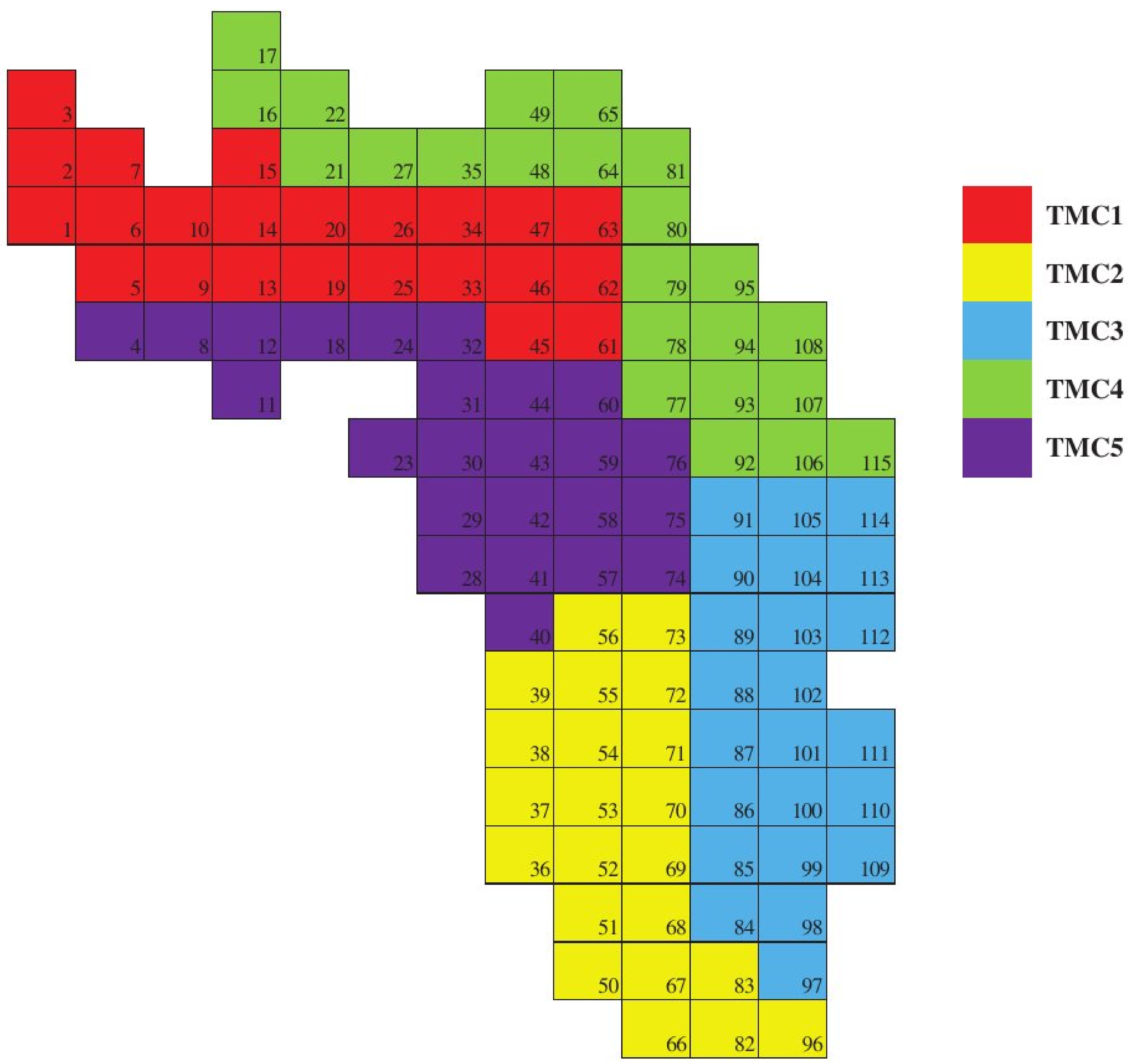

Figure 3: TMCs obtained by the partitioning algorithm [46]

Since the results of the partitioning algorithm obtained in Table 4 meet the technological requirements, they actually represent the values of required technological vectors. Accordingly, the *ADM* matrix is created in the following form:

$$ADM = \left|z_{ij}\right|_{5\times 4} = \begin{vmatrix} A/C & c_1^{teh} & c_2^{teh} & c_3^{teh} & c_4^{teh} \\ TMC_1 & 169601 & 39.13 & 0.325 & 1.198 \\ TMC_2 & 170241 & 30.72 & 0.292 & 0.418 \\ TMC_3 & 172156 & 29.39 & 0.381 & 0.820 \\ TMC_4 & 172859 & 62.78 & 0.169 & 0.928 \\ TMC_5 & 177388 & 48.16 & 0.267 & 1.262 \end{vmatrix} \quad (25)$$

The efficiency evaluation of the algorithm is performed through the following steps:

Step 1: The detailed calculation process of the weights of criteria by the SPC method over the ADM matrix is demonstrated Step by Step.

Step 1.1: The elements of an artificial decision matrix are represented as follows:

$$DM = \left|x_{ij}\right|_{5\times 4} = \begin{vmatrix} A/C & C_1 & C_2 & C_3 & C_4 \\ A_1 & 169601 & 39.13 & 0.325 & 1.198 \\ A_2 & 170241 & 30.72 & 0.292 & 0.418 \\ A_3 & 172156 & 29.39 & 0.381 & 0.820 \\ A_4 & 172859 & 62.78 & 0.169 & 0.928 \\ A_5 & 177388 & 48.16 & 0.267 & 1.262 \end{vmatrix} \quad (26)$$

Step 1.2: The calculation of the symmetry point of each criterion is represented in Table 5.

Table 5: Symmetry point of criterion

Criterion	$Min\ min\left\{x_{ij}\right\}$	$Max\ max\left\{x_{ij}\right\}$	Symmetry point $\frac{min\left\{x_{ij}\right\} + max\left\{x_{ij}\right\}}{2}$
C_1	169601	177388	173494.5
C_2	29.390	62.780	46.085
C_3	0.169	0.381	0.275
C_4	0.418	1.262	0.840

Step 1.3: The resulting matrix of absolute distances is as follows:

$$D = \begin{vmatrix} A/C & C_1 & C_2 & C_3 & C_4 \\ A_1 & |169601 - 173494.5| & |39.13 - 46.085| & |0.325 - 0.275| & |1.198 - 0.840| \\ A_2 & |170241 - 173494.5| & |30.72 - 46.085| & |0.292 - 0.275| & |0.418 - 0.840| \\ A_3 & |172156 - 173494.5| & |29.39 - 46.085| & |0.381 - 0.275| & |0.820 - 0.840| \\ A_4 & |172859 - 173494.5| & |62.78 - 46.085| & |0.169 - 0.275| & |0.928 - 0.840| \\ A_5 & |177388 - 173494.5| & |48.16 - 46.085| & |0.267 - 0.275| & |1.262 - 0.840| \end{vmatrix} \quad (27)$$

$$D = \begin{vmatrix} A/C & C_1 & C_2 & C_3 & C_4 \\ A_1 & 3893.5 & 6.955 & 0.050 & 0.358 \\ A_2 & 3253.5 & 15.365 & 0.017 & 0.422 \\ A_3 & 1338.5 & 16.695 & 0.106 & 0.020 \\ A_4 & 635.5 & 16.695 & 0.106 & 0.088 \\ A_5 & 3893.5 & 2.075 & 0.008 & 0.422 \end{vmatrix} \quad (28)$$

Step 1.4: Elements x_{11} and x_{12} of the matrix of the moduli of symmetry are calculated in the following way:

$$x_{11} = \frac{\frac{3893.5 + 3253.5 + 1338.5 + 635.5 + 3893.5}{5}}{169601} = 0.0153;$$

$$x_{12} = \frac{\frac{6.955 + 15.365 + 16.695 + 16.695 + 2.075}{5}}{39.13} = 0.2953 \tag{29}$$

Analogically, we obtain values for the remaining elements, and the matrix of the moduli of symmetry is as follows:

$$R = \begin{vmatrix} A/C & C_1 & C_2 & C_3 & C_4 \\ A_1 & 0.0153 & 0.2953 & 0.1766 & 0.2187 \\ A_2 & 0.0153 & 0.3762 & 0.1966 & 0.6268 \\ A_3 & 0.0151 & 0.3932 & 0.1507 & 0.3195 \\ A_4 & 0.0151 & 0.1841 & 0.3396 & 0.2823 \\ A_5 & 0.0147 & 0.2400 & 0.2150 & 0.2076 \end{vmatrix} \tag{30}$$

Step 1.5: The modulus of the symmetry of the first criterion C_1 is calculated as follows:

$$q_{11} = \frac{0.0153 + 0.0153 + 0.0151 + 0.0151 + 0.0147}{5} = 0.0151 \tag{31}$$

The remaining moduli of symmetry are calculated in a similar way, and values are represented by the following vector Q:

$$Q = |q_{1\,5}|_{1\times 5} = \begin{vmatrix} 0.0151 & 0.2978 & 0.2157 & 0.3310 \end{vmatrix} \tag{32}$$

Step 1.6: Weight of the first criterion C_1 is:

$$w_1 = \frac{0.0151}{0.0151 + 0.2978 + 0.2157 + 0.3310} = 0.0176 \tag{33}$$

The weights of criteria for the artificial MCDM problem are shown in Table 6, and Fig. 4.

Table 6: The weights of criteria

	C_1	C_2	C_3	C_4
Weight-w_j	0.0176	0.3464	0.2509	0.3851

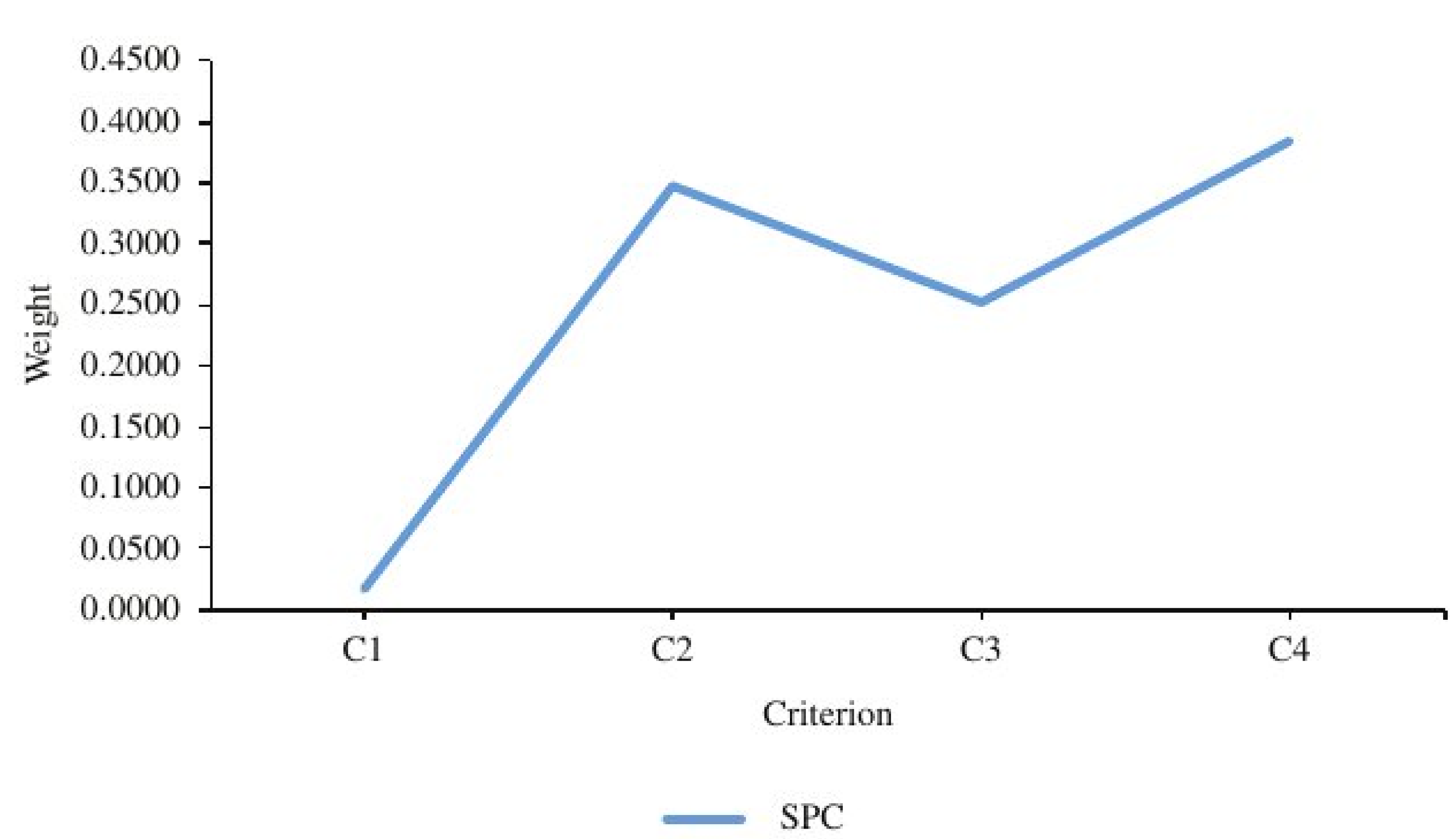

Figure 4: Weights of criteria calculated by the SPC method

Step 2: The values of criteria weights calculated by the SPC method are presented in Table 13. The weights of the optimal criteria are shown in Table 7, and Fig. 5.

Table 7: The weights of the technological and optimal criteria

Criterion Weight	c_1^{teh} w_1^{teh}	c_2^{teh} w_2^{teh}	c_3^{teh} w_3^{teh}	c_4^{teh} w_4^{teh}
Vector W^{teh}	0.0176	0.3464	0.2509	0.3851
Criterion Weight	c_1^{opt} w_1^{opt}	c_2^{opt} w_2^{opt}	c_3^{opt} w_3^{opt}	c_4^{opt} w_4^{opt}
Vector W^{op}	0.25	0.25	0.25	0.25

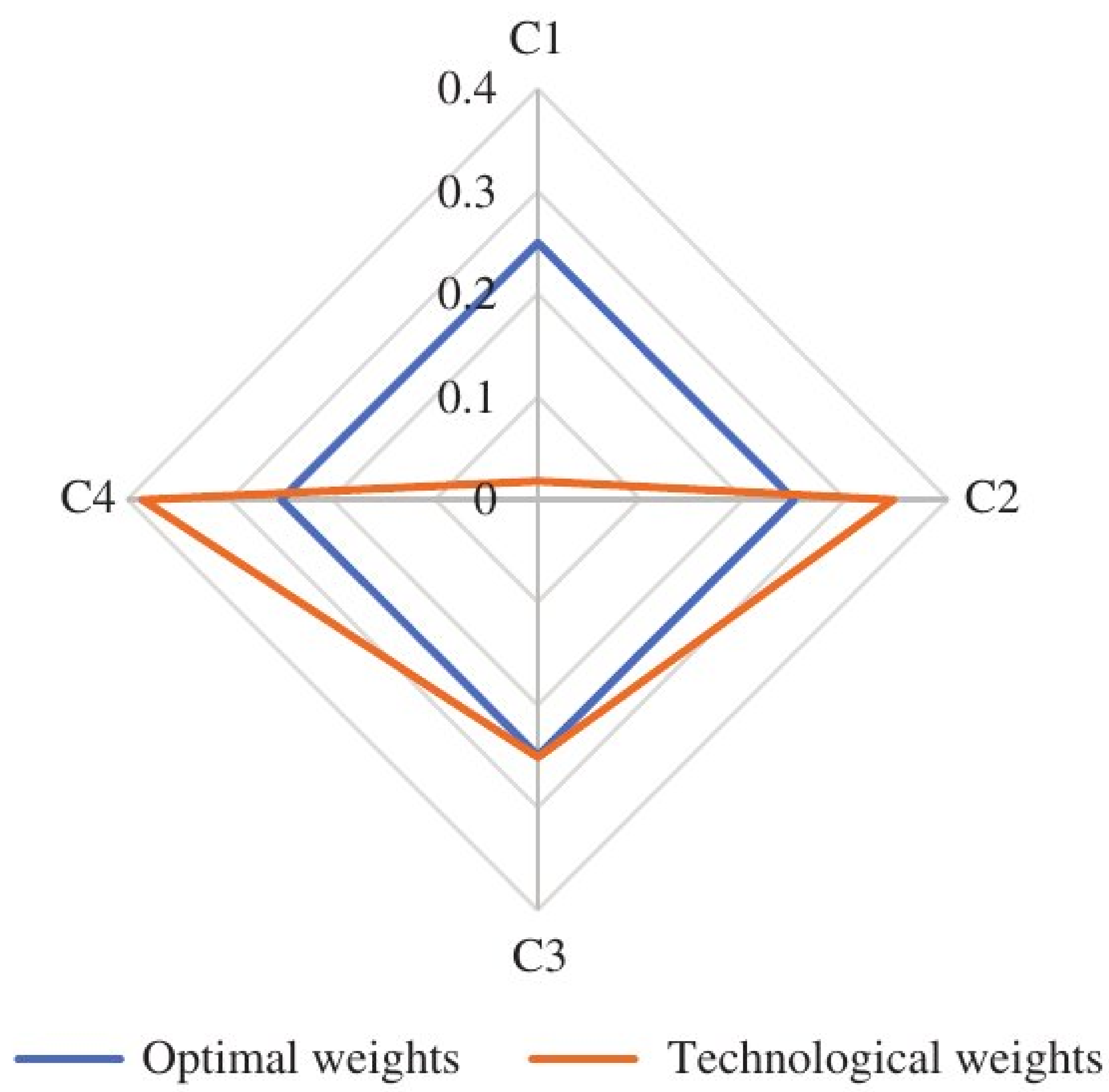

Figure 5: Optimal and technological weights of criteria

Step 3: The entropy of the calculated weights and optimal weights is computed as follows:

$$H^{teh} = -\sum_{j=1}^{K} w_j^{teh} \cdot log\left(w_j^{teh}\right) = (-0.0176 \cdot log\,(0.0176)) + (-0.3464 \cdot log\,(0.3464)) + (-0.2509 \cdot log\,(0.2509)) + (-0.3851 \cdot log\,(0.3851)) = 0.5006 \quad (34)$$

$$H^{opt} = -\sum_{j=1}^{K} w_j^{opt} \cdot log\left(w_j^{opt}\right) = (-0.25 \cdot log\,(0.25)) + (-0.25 \cdot log\,(0.25)) + (-0.25 \cdot log\,(0.25)) + (-0.25 \cdot log\,(0.25)) = 0.6021 \quad (35)$$

Step 4: The efficiency of the algorithm (*EOA*) is calculated as:

$$EOA = \frac{H^{teh}}{H^{op}} \times 100\% = \frac{0.5006}{0.6021} \times 100\% = 83.15\% \quad (36)$$

The mineral deposit used in the example can be treated as a very difficult environment for the process of partitioning, with respect to the deposit's characteristics. The space distribution of ore grades, ore tonnage, and the shape of the deposit are extremely irregular. Bearing in mind the very rigorous technological requirements, it can be said that the mineral deposit partitioning problem is a complex task. Despite this, the algorithm succeeded in dividing the mineral deposit with *EOA* of 83.15%, i.e., the efficiency of the algorithm is at a very high level.

4 A Comparison of the Objective Weighting Methods for Determining the Weights of Criteria

This section is related to the comparative analysis to show the capability of the SPC method to define veritable weights of criteria. The proposed SPC method is compared with the most frequently applied approaches for determining criteria weights, such as the CRITIC [7,8,10], Entropy [7,8,50], Standard Deviation [7,10,50] and MEREC [13,14,51] methods. The CRITIC, Entropy, Standard Deviation and MEREC methods used linear normalized decision matrix data to compute the weights of the criteria.

Beside the SD method, the CRITIC and Entropy methods are conventional and widely implemented objective weighting methods that are commonly used for determining criteria weights. These well-known methods are quite simple and easy to understand for decision-makers. Although they require a relatively high level of mathematical calculation, their application is growing constantly. Consequently, weight coefficients obtained by these methods are very reliable and stable. An enormous number of authors have also applied these two methods as a benchmark to compare their new developed methods. Due to these significant specifics, we selected the CRITIC and Entropy methods for comparison with our novel SPC method. In contrast to these usual methods, we have utilized a relatively new objective weighting method for comparison known as the MEREC method. In that way, we obtained an effective verification and validation of our new proposed method.

Our improvement of the CRITIC method involves changing the normalization procedure of the input data. The CRITIC method is quite sensitive to the application of different normalization techniques. The high impact of the normalization of input data is reflected in the final rank of the weight coefficients. Instead of the standard linear max-min normalization technique used in the CRITIC method, we applied linear sum procedure and obtained much better results for criteria weights. Because of these limitations, several authors have developed new modified and extended versions of the classical CRITIC method to improve the final values of weight coefficients [1,2,52]. As can be seen from Fig. 6 and Tables 9 and 10, the CRITIC method has the lowest degree of correlation with all the applied objective weighting methods.

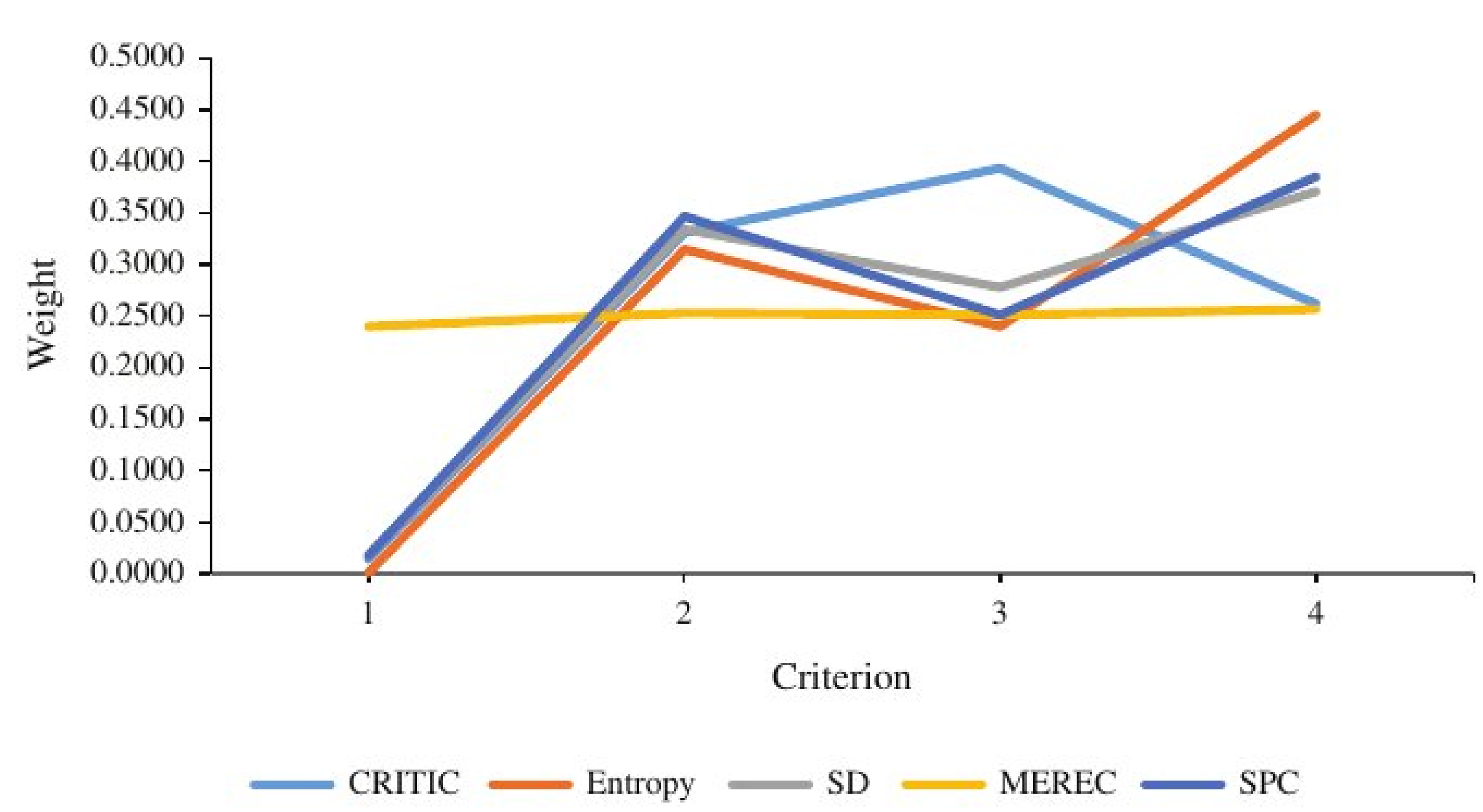

Figure 6: Weights of criteria calculated by methods of comparison

Weights of criteria calculated by these methods are shown in Table 8 and in Fig. 6.

Table 8: Weights of criteria calculated by comparative methods

Weight	CRITIC	Entropy	SD*	MEREC	SPC (proposed)
w_1	0.0141	0.0009	0.0181	0.2399	0.0176
w_2	0.3309	0.3146	0.3338	0.2526	0.3464
w_3	0.3932	0.2397	0.2777	0.2511	0.2509
w_4	0.2618	0.4448	0.3705	0.2564	0.3851

Note: *SD–Standard Deviation.

The Pearson coefficient was used to define the degree of correlation between the weights of criteria computed by the CRITIC, Entropy, Standard Deviation, MEREC and SPC methods. The results of the comparative analysis are presented in Table 9.

Table 9: Comparative analysis results–correlation coefficients

Coefficient	CRITIC	Entropy	SD*	MEREC	SPC
CRITIC		0.6974	0.8424	0.7965	0.7830
Entropy	0.6974		0.9696	0.9885	0.9805
SD*	0.8424	0.9696		0.9915	0.9937
MEREC	0.7965	0.9885	0.9915		0.9869
SPC	0.7830	0.9805	0.9937	0.9869	

Note: *SD–Standard Deviation.

Average correlation coefficients for each method of criteria weighting are shown in Table 10.

Table 10: Average values of comparative analysis parameters for criteria weights

Method	Pearson coefficient
CRITIC	0.7798
Entropy	0.9090
SD*	0.9493
MEREC	0.9409
SPC	0.9360

Note: *SD–Standard Deviation.

Rank ordering of the weights of criteria for different criteria weighting methods is presented in Table 11 and Fig. 7.

Table 11: Scores for rank ordering of the weights of criteria

		Criteria weighting method			
Criterion	CRITIC Rank	Entropy Rank	SD* Rank	MEREC Rank	SPC Rank
C_1	4	4	4	4	4
C_2	2	2	2	2	2
C_3	1	3	3	3	3
C_4	3	1	1	1	1

Note: *SD–Standard Deviation.

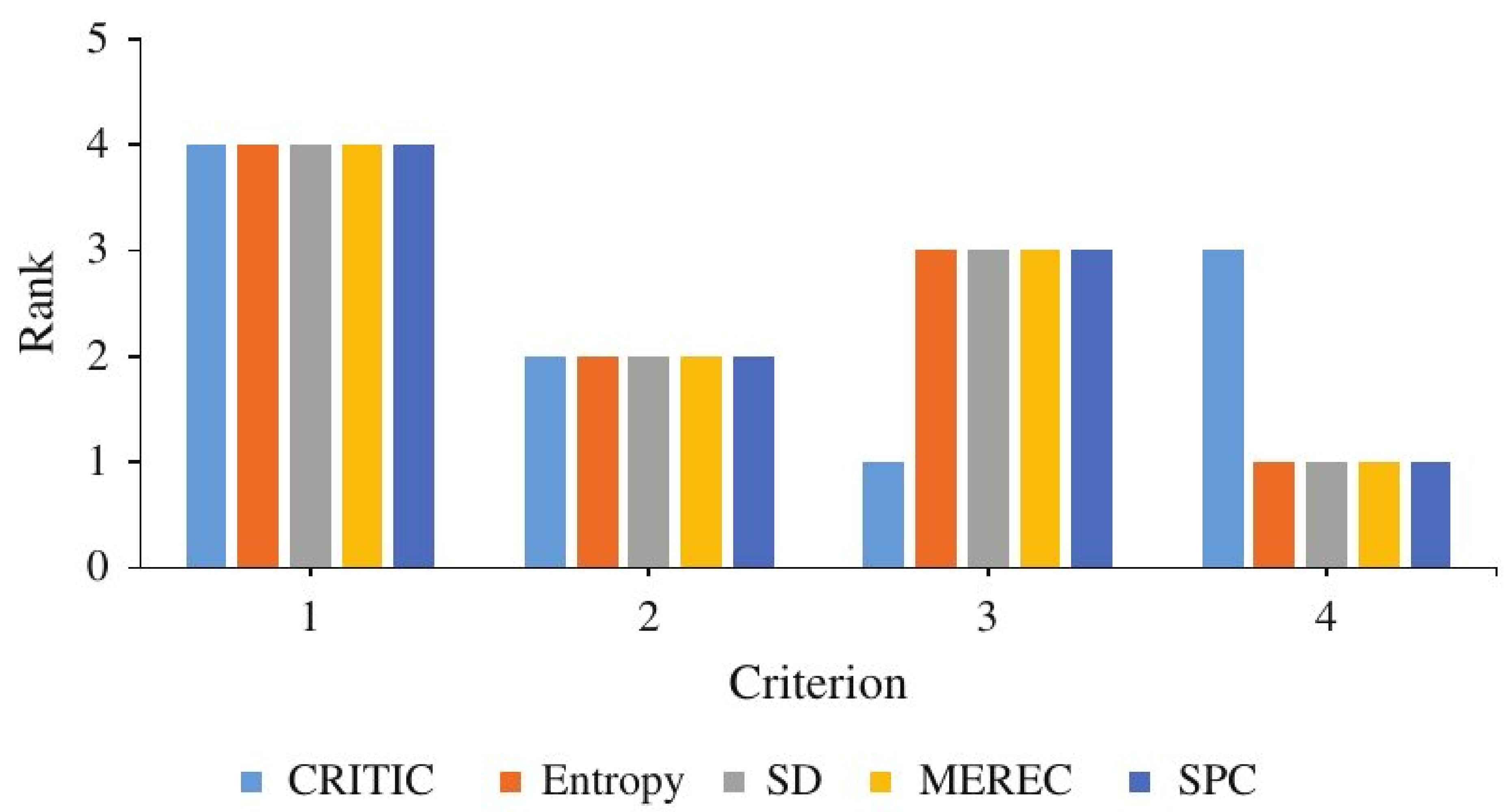

Figure 7: Rank order of the weights of criteria

The same approach is used to make a comparative analysis for the rank ordering of the weights of criteria. The comparative analysis results are presented in Table 12.

Table 12: Comparative analysis results for criteria weights rank ordering

Coefficient	CRITIC	Entropy	SD*	MEREC	SPC
CRITIC		0.2000	0.2000	0.2000	0.2000
Entropy	0.2000		1.0000	1.0000	1.0000
SD*	0.2000	1.0000		1.0000	1.0000
MEREC	0.2000	1.0000	1.0000		1.0000
SPC	0.2000	1.0000	1.0000	1.0000	

Note: *SD–Standard Deviation.

The average correlation coefficients for each method of criteria rank ordering are shown in Table 13.

Table 13: Average values of comparative analysis parameters for criteria rank ordering

Method	Pearson coefficient
CRITIC	0.2000
Entropy	0.8000
SD*	0.8000
MEREC	0.8000
SPC	0.8000

Note: *SD–Standard Deviation.

The values shown in Table 9 indicate the existence of a strong correlation between the weights of criteria assessed by the SPC method and the weights assessed by the Entropy, Standard Deviation and MEREC methods. There is a slightly lower correlation between the SPC, Entropy, SD and MEREC methods on one side compared with the CRITIC method on the other. The average Pearson coefficient of correlation (0.9360; see Table 10) shows that the SPC method stands shoulder to shoulder with the other considered methods.

A comparative analysis using the criteria rank ordering is a more rigorous approach than the previous analysis. In such comparison circumstances, the SPC method showed very acceptable results. The correlation coefficient with the Entropy, Standard Deviation and MEREC methods is extremely high (1.000; see Table 12), while with the CRITIC method it is low (0.2000; see Table 12). The average coefficient of correlation of 0.8000 indicates the high level of applicability of the SPC method.

According to the above discussions, we can say that the SPC method is very capable of assessing the weights of criteria in MCDM problems. Furthermore, all the parameters of comparison showed that the developed method is very reliable.

The efficiency of the algorithm (*EOA*) by other objective weighting methods that are used for comparison analysis is calculated using the Eq. (24) and the results obtained are shown in Table 14 and Fig. 8.

Table 14: Efficiency of the algorithm (*EOA*) by all objective weighting methods

Method	CRITIC	Entropy	SD*	MEREC	SPC
EOA (%)	82.52	77.41	83.85	99.98	83.15

The efficiency of the algorithm determined by all the applied objective weighting methods is shown in Table 14 and Fig. 8. From the obtained results, it is clearly visible that all methods have either a high and or high *EOA* value, according to Table 1. The MEREC method with 99.98% *EOA* indicates superiority over the other methods, although the weights of criteria are not graphically matched to other methods as can be seen from Fig. 6. However, the rank ordering of the weights of criteria obtained by the MEREC method has an extremely high correlation with other methods. The SPC method demonstrated the very high value of 83.15% *EOA* relative to the other methods (CRITIC 82.52%, Entropy 77.41% and SD 83.85% of *EOA*). Obviously, SPC is completely competitive with all methods and absolutely applicable in solving such a complex problem providing verification and validation of the partitioning algorithm.

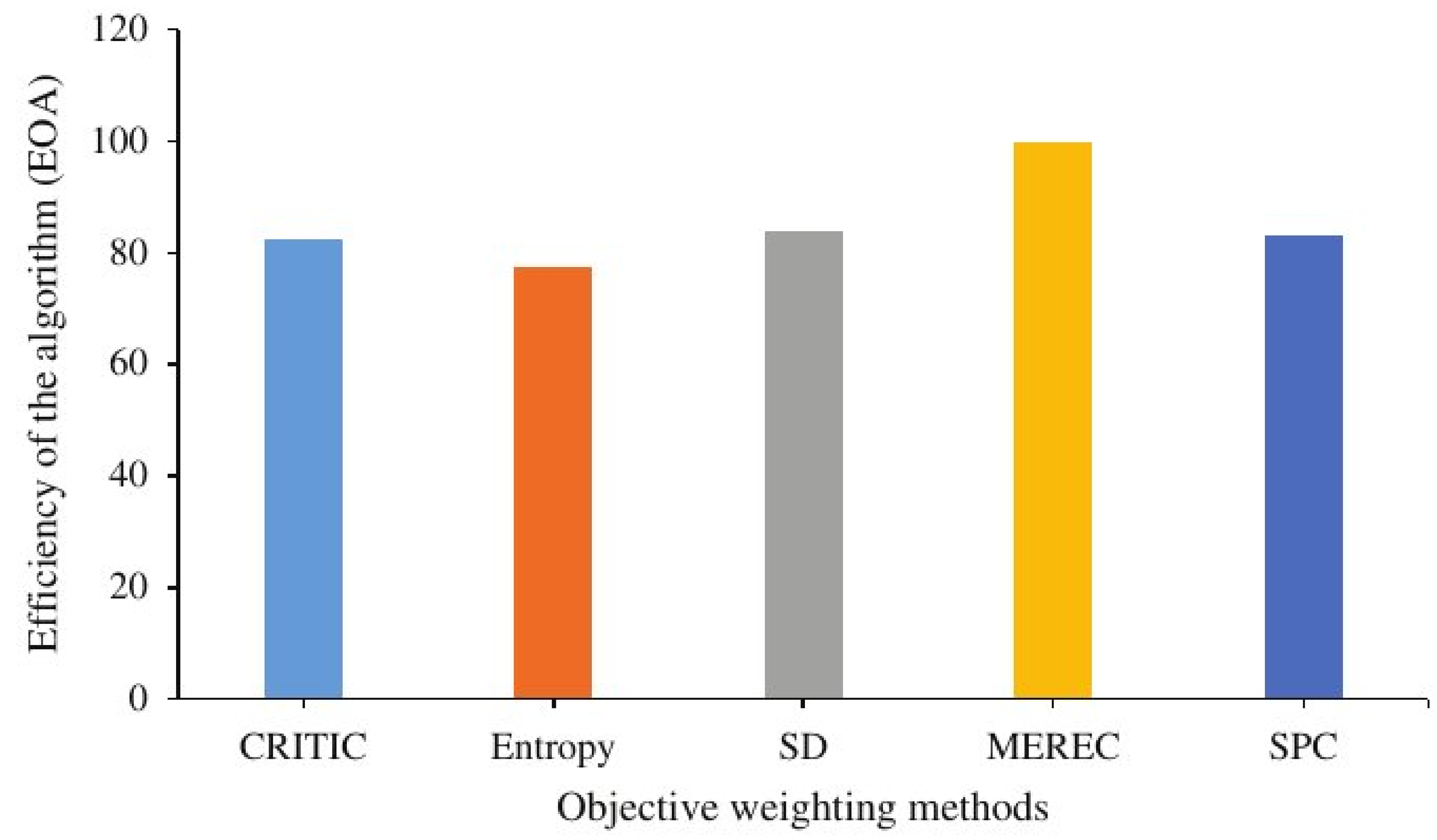

Figure 8: Graphical review of the efficiency of the algorithm

From the extensive comparison analysis, we can highlight two main advantages of the new proposed SPC method. The first advantage is reflected in the fact that this method illustrates a new objective weighting method for defining the weights of criteria. Each MCDM technique should increase objectivity during the decision-making process. Since the attribute importance plays a very dominant role in the decision-making process, this proposed method helps decision-makers to obtain a more objective and reliable final rank of alternatives. The second advantage of the SPC method is expressed through the efficiency evaluation of the mineral deposit partitioning algorithm. This approach presents the novel mechanism for estimating the performance of every partitioning algorithm. It means that a uniform distribution of weighted coefficients indicates the high efficiency of the algorithm. As a very flexible and understandable approach, the SPC method can be extensively incorporated with conventional MCDM techniques for solving different problems. Certainly, the new proposed SPC method can be successfully implemented in numerous spheres of science. In a domain of technical sciences, this method can be applied in electrotechnical, mechanical, civil, traffic, energy and many other sectors of engineering. Beside this wide application in engineering fields, the SPC method can be efficiently used in various areas, such as economic fields, medical disciplines, social problems and even in political sciences.

5 Conclusions, Limitations and Future Research

Every real-life problem contains an important element faced through a decision-making challenge. Sometimes decision-makers must react very quickly in a short period of time and under uncertain conditions. Mining engineers are constantly faced with a huge number of challenges. Every investor and mining company management tends to create an optimal production plan as quickly as possible to begin mining activities. This optimal production plan is recognized as a crucial activity if mining companies are to make a profit in a very risky and volatile environment. The new proposed SPC method can be very useful for mining engineers in forming an optimal production plan. Although this method represents the verification of a previously developed partitioning algorithm, it can be applied as a tool for partitioning the mineral deposits in future studies.

The weights of criteria have a significant influence on the solving of multi-criteria problems. Besides the alternatives ranking method, the objective evaluation of criteria weights is also recognized as a very important activity. Excluding the subjectivity related to the preference of criteria should help in selecting the best alternative, i.e., in making the best decision. For that purpose, we have developed an objective method called Symmetry Point of Criteria (the SPC method). Basically, this method measures the absolute distance between the criterion value of a range of alternatives and the symmetry point of the criterion. The symmetry point represents the midpoint between two extreme criterion values (*min* and *max* value of the criterion). Every objective method can result in different values of criteria for individual decision-making problems but obtained values should be consistent as much as possible. Comparative analysis between the novel SPC method and the CRITIC, Entropy, Standard Deviation and MEREC methods shows that our method is capable of determining the criteria weights in a very efficient way. The average Pearson correlation coefficient of criteria rank ordering is 0.8000. The real-life applicability of the SPC method is demonstrated in the case of the mineral deposit multi-criteria partitioning algorithm. The efficiency of the algorithm depends directly on the values of the weights of criteria. If the calculated weights are mainly uniformly distributed, then the efficiency of the algorithm is higher.

The limitations of the proposed method are as follows. First of all, a hypothetical example is used to test the efficiency of the mineral deposit partitioning algorithm by the SPC method. Future research should be focused on implementing a real-life case study for testing the performance of the above-mentioned partitioning algorithm. Beside this obvious shortcoming, the non-inclusion of uncertainty theories into the initial decision-making matrix with input data is recognized as another limitation of the proposed method. Future research must include many different types of fuzzy, neutrosophic and intuitionistic numbers, creating an uncertain environment, and the final ranking of alternatives will be obtained under more reliable conditions. It is possible to interpret the limitations of the case study as follows. Firstly, only the basic attributes (four technological criteria) that characterize a mineral deposit are considered for the partitioning algorithm of the mineral deposit in this case study. In future works, there is a possibility to increase the number of criteria and test the efficiency of the partitioning algorithm by this proposed SPC method. Secondly, one of the key attributes that must meet the technological requirements is ore grade. Since the standard deviation is used to calculate ore grade, involving fuzzy numbers in the process of calculating the standard deviation of the ore grade is a very difficult and complex task. Future work can be directed towards overcoming this problem in the calculation process of standard deviation under a fuzzy environment by developing software for that purpose.

If we take into consideration that many real-life multi-criteria decision-making problems are burdened by uncertainties, then future research will be directed to including them in the SPC method. We will also explore how the developed method behaves in such environments. An extended version of our paper can be performed by introducing the stochastic diffusion process for the purpose of describing the features of some criteria. In that way, we can develop a dynamic model that is capable of solving more complex problems in the real world. Another variant of upgrading the paper should be related to integrating the SPC method with other MCDM processes. Certainly, there is potential to combine our developed SPC method with other subjective and objective weighting methods producing a new hybrid approach for calculating the weights of criteria.

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