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DEVELOPMENT OF INTEGRATED FUZZY MODEL FOR MINE MANAGEMENT OPTIMIZATION

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Abstract

Technoeconomic, environmental and safety criteria generally affect the management of metallic and non-metallic mining operations. The first basic question that needs to be addressed when planning ore mining is which methods are adequate and what is the optimal mining technology? Due to the complex geologic framework of ore deposits, geological exploration has rendered synonymous the inherent uncertainties, vagueness, and inaccuracies. As a result, subjective evaluation by engineers and expert experience have become increasingly important. Given that the natural language used by miners and geologists is most suited for relaying knowledge and expressing opinions, the paper tests a fuzzy optimization methodology that uses linguistic variables. Consequently, extent analysis is applied to fuzzy AHP by means of triangular fuzzy numbers to arrive at a decision about the optimal mining technology. The entire procedure constitutes an integrated mine management system, which will contribute to sustainable production in the future. A case study to which the model was applied is presented in the paper.

Key words: mining technology, expert evaluation, triangular fuzzy numbers, multicriteria decision making

Introduction. Mine management optimization, and thus the importance of mining activity planning, stem from the fact that production costs are measured

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in enormous amounts of money. This industry hires a large number of miners, who require favourable working conditions. Additionally, ore deposits are largely found at large depths, causing certain mine management problems that need to be examined. On the one hand, this includes excavation costs, extractable ore reserves, and ore depletion during the course of mining. On the other hand, a safe mining technology and a healthy work environment need to be ensured for miners.

The paper describes a model used to arrive at an optimal solution for selecting the appropriate mining technology. Given that decision making in the case of complicated geological conditions associated with ore deposits requires "multidimensional" opinions, where experts are required to use logic, knowledge, intuition and experience, the proposed methodology addresses all these issues and fuzzy logic is its integral part [1,2]. When fuzzy logic is used for multicriteria decision making, the criteria are described by linguistic variables and represented in the form of triangular fuzzy numbers (membership functions: “L, S, D”). On the one hand, the criteria are analyzed and evaluated using a scale of relative importance. On the other hand, mathematical optimization calculations are performed applying the fuzzy analytic hierarchy process FAHP [3–5] to select the optimal mining technology.

The great advantage of fuzzy logic and heuristic methods need to be emphasized in connection with geological exploration of ore deposits. It is primarily related to the complexity of geologic structures and the physical and mechanical properties of rocks. In such circumstances, engineer experience is especially important. Several interesting studies addressing the fuzzy logic concept in mining are reported in [6–8].

The integrated fuzzy model for mining technology optimization was applied to a real case study, bauxite deposit L-29C of the Bešpelj mine near Jajce in the Republic of Serbia, Bosnia and Herzegovina, which is the focus of the paper.

Study area and geology. The Bešpelj bauxite mine is located on a spacious karst plateau, on the right bank of the Vrbas River. It is at a beeline distance of 10 km from the town of Jajce to the north. The longer axis of deposit L-29C trends west-to-east (140 m), and the shorter axis is perpendicular (15 m long, tapering out to 1 m in the east). The ore body is vertical. The deposit is close to the land surface, at a depth of about 100 m, and the quality of the bauxite ore is high.

The geologic framework of the deposit was defined based on exploratory activities, largely data collected from exploratory boreholes. The tectonic relationships are highly complex. The deposit is in a carbonate rock environment, including underlying and overlying limestones. The underlying limestones are solid homogeneous rocks, with numerous fractures and a number of fault zones. Expert experience suggested potential problems for certain types of mining operations. Additionally, also based on experience and the physical and mechanical properties of bauxite, underground working areas required supporting in places of well-developed caverns. The overlying rocks are Senonian limestones, with numerous fracture systems, similar to those in the underlying rocks because of their common structural evolution. They, too, are well stratified and the thickness of the strata mostly ranges from 0.1 to 2 m. Geomechanical properties were particularly investigated in the ore deposit zone, where tectonic transport was most distinct, and found to be rather unfavourable. The most adverse area for underground mining operations is the zone of contact between the overlying and underlying limestones, and especially within the ore deposit and its immediate environment. Experience indicates a danger of limestone blocks detaching from the ceilings of mining rooms. Figure 1 shows the zone of the ore deposit described in the paper.

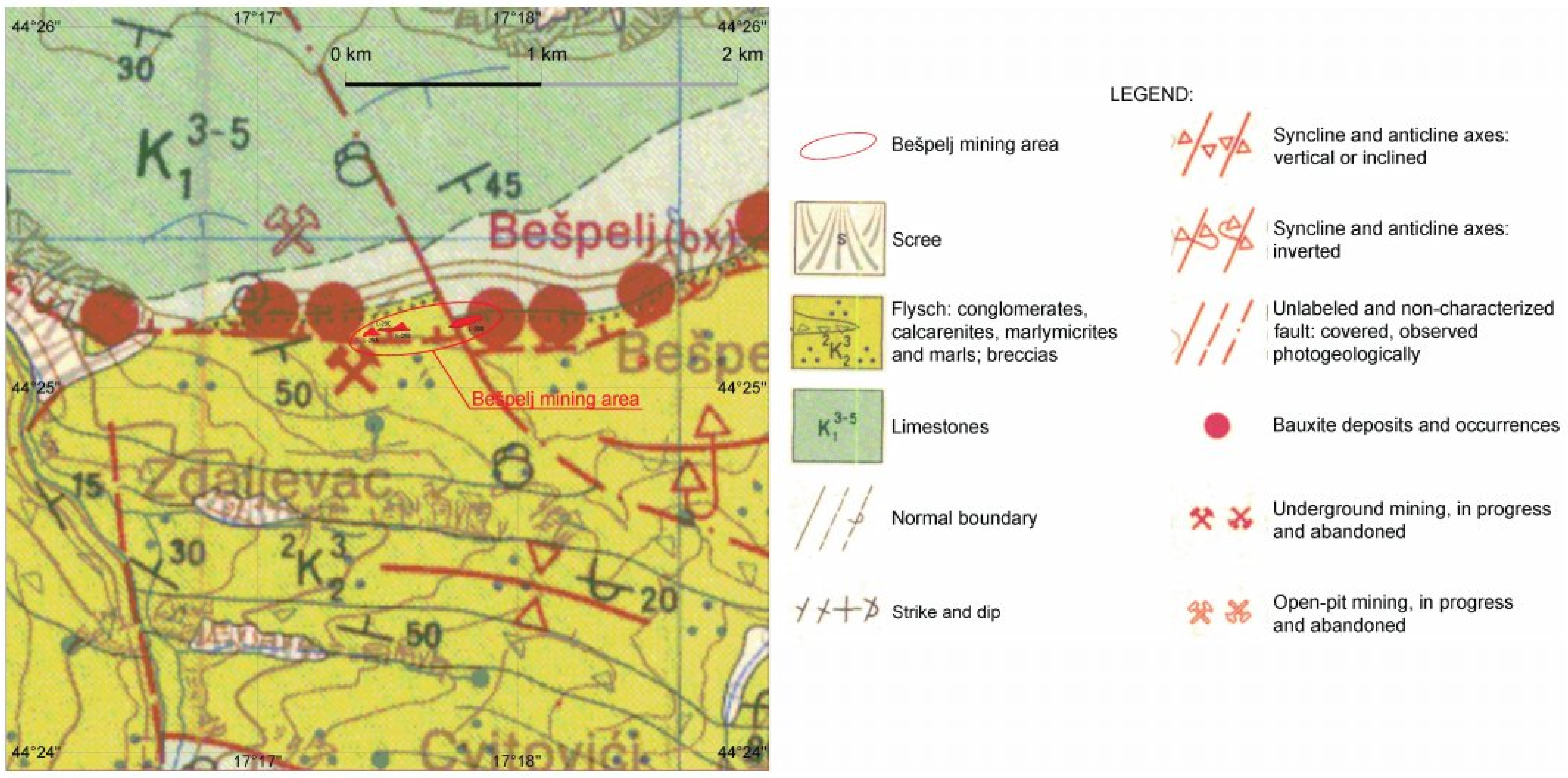

Fig. 1. Geological map of the study area (modified [9])

Methodology. As mentioned above, the FAHP approach was followed to arrive at a decision on the optimal mining technology in the case study. An overview is provided below. More detailed descriptions of the application of the methodology in geology and mining are available in [7, 10]. At the beginning, criteria matrices are created relative to alternative solutions, using a fuzzified scale of relative importance and triangular fuzzy numbers [3, 5, 11]. Element identification serves to formulate a question for the expert who is examining the problem: "Is a criterion better for pairwise comparison in the matrix, and to what extent?" Proper matrix generation of all the criteria relative to alternative solutions plays a major role in optimal decision making. The solutions are optimized by fuzzy extent analysis, applying the FAHP method described in [3]. The optimization procedure and decision making are presented in seven steps [12].

Step 1. Mathematical optimization begins with predefined criteria that affect the selection of one of several alternatives. Then a matrix of criterion X is constructed with fuzzy numbers assigned by the decision maker (expert) using the FAHP scale.

Step 2. An extent analysis is undertaken of all the elements in the matrix from the previous step. This results in m values of step analyses for each element of set X: $M_{g_i}^1, M_{g_i}^2, \ldots, M_{g_i}^m$, $i = 1, 2, \ldots, n$, where all $M_{g_i}^j$, $j = 1, 2, \ldots, m$ are triangular fuzzy numbers.

Then, taking into account the membership function of the triangular fuzzy number, the fuzzy synthetic extent is calculated using the following equation:

$$S_i = \sum_{j=1}^{m} M_{g_i}^j \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^j \right]^{-1} = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} s_j, \sum_{j=1}^{m} d_j \right) \otimes \left(\frac{1}{\sum\limits_{i=1}^{n} d_j}, \frac{1}{\sum\limits_{i=1}^{n} s_j}, \frac{1}{\sum\limits_{i=1}^{n} l_j} \right).$$

Step 3. The degree of possibility of two triangular fuzzy numbers is determined using the fuzzy number comparison approach:

$$V\left(M_1 \geq M_2\right) = \sup_{x \geq y} \left[\min\left(\mu_{M_1}(x), \mu_{M_2}(y) \right) \right].$$

If there are such pairs (x, y) that $x \geq y$ and $\mu_{M_1}(x) = \mu_{M_2}(y) = 1$, then $V\left(M_1 \geq M_2\right) = 1$. Given that M_1 and $M_2$are convex triangular fuzzy number, it follows that

$$\begin{aligned} & V\left(M_1 \geq M_2\right) = 1 \quad \text{if} \;\; s_1 \geq s_2 \\ & V\left(M_2 \geq M_1\right) = hgt\left(M_1 \cap M_2\right) = \mu_{M_1}(c) \begin{cases} 1, & \text{if} \;\; s_2 \geq s_1 \\ 0, & \text{if} \;\; l_1 \geq d_2 \\ \dfrac{l_1 - d_2}{(s_2 - d_2) - (s_1 - l_1)}, & \text{other} \end{cases} \end{aligned}$$

where c is the ordinate of the highest intersection in point C between membership functions μ_{M_1} and μ_{M_2}.

Both $V\left(M_1 \geq M_2\right)$ and $V\left(M_2 \geq M_1\right)$ are required to compare M_1 and M_2.

The degree of possibility of a convex triangular fuzzy number, so that it is greater than k of convex fuzzy number M_i, where $i = 1, 2, \ldots, k$, can be defined as

$$\begin{aligned} V\left(M \geq M_1, M_2, \ldots, M_k\right) &= V\left[\left(M \geq M_1\right) \wedge \left(M \geq M_2\right) \wedge \cdots \wedge \left(M \geq M_k\right) \right] \\ &= \min V\left(M \geq M_i\right). \end{aligned}$$

Summing everything up yields

$$c\text{'}\left(A_i\right) = \min V\left(S_i \geq S_k\right), \qquad k = 1, 2, \ldots, n; k \neq i.$$

Step 4. Weight priority vectors are defined as

$$W' = \left(c'\left(A_1\right), c'\left(A_2\right), \ldots, c'\left(A_n\right)^T \right) \;\; \text{where} \;\; A_i\left(i = 1, 2, \ldots, n\right).$$

Step 5. The final weights are calculated applying a matrix algebra approach, namely the additive normalization method [11]. The result is a normalized weight priority vector in the form of a non-fuzzy number, whose maximum value is 1: $W = \left(c\left(A_1\right), c\left(A_2\right), \ldots, c\left(A_n\right)^T\right)$.

Step 6. The alternatives are compared against each criterion individually. Thus, new matrices are defined and then the weight priority vectors determined, as described in Steps 1 through 5.

Step 7. The final weights of the alternatives are determined by multiplying the "weights" from the criteria matrix by the "weights" obtained under Step 6. The highest "weight" value represents the optimal alternative [13,14].

Results. The following criteria were considered in the case of the L-29 C bauxite deposit at Bešpelj Mine: K_1 – operating expenses, K_2 – safe and healthy work environment, K_3 – extractable ore reserves, and K_4 – coefficient of ore depletion. The considered mining technology options, based on the natural setting, included: A_1 – block caving, A_2 – sublevel caving, A_3 – cutting and filling, and A_4 – room and pillar mining.

The proposed fuzzy optimization methodology was applied considering the above criteria and options. The calculations were made in a specially developed FUZZY-GWCS program [10], and the input elements were numerical values of linguistic variables. Table 1 shows the criteria matrix values and the calculated weight coefficients. The scores and weights are represented by triangular fuzzy numbers.

Table 2 shows the scores of the alternatives relative to each criterion, as well as the weight coefficients. Additionally, triangular fuzzy numbers are used to show the comparison of all alternatives relative to set criteria, as well as calculated weights.

The final scores of all the alternatives, in the form of triangular fuzzy numbers, were calculated under Step 5, based on Eq. 5, and then the final "weights" of the alternatives were derived in the form of non-fuzzy numbers. The optimization indices shown in Table 3 resulted from Step 6.

Based on interpreted results, the alternative with the largest "weight" re-

Table 1

Criteria analysis

Criterion	K1			K2			K3			K4			Weight coefficients		
K1	1	1	1	4	5	6	4	5	6	6	7	8	0.375	0.538	0.774
K2	0.17	0.2	0.25	1	1	1	2	3	4	4	5	6	0.179	0.275	0.415
K3	0.17	0.2	0.25	0.25	0.33	0.5	1	1	1	2	3	4	0.085	0.135	0.212
K4	0.13	0.14	0.17	0.17	0.2	0.25	0.25	0.33	0.5	1	1	1	0.038	0.051	0.071

Table 2

Analysis of alternatives relative to criteria

Criterion	A1			A2			A3			A4			Weight coefficients		
K1															
A1	1	1	1	0.33	0.5	1	0.25	0.3	0.5	1	2	3	0.086	0.176	0.372
A2	1	2	3	1	1	1	1	2	3	1	2	3	0.134	0.324	0.677
A3	2	3	4	0.33	0.5	1	1	1	1	3	4	5	0.212	0.394	0.744
A4	0.33	0.5	1	0.33	0.5	1	0.2	0.25	0.3	1	1	1	0.062	0.104	0.225
A1	1	1	1	0.33	0.5	1	0.25	0.3	0.5	1	2	3	0.085	0.165	0.332
A2	1	2	3	1	1	1	3	4	5	2	3	4	0.231	0.435	0.786
A3	2	3	4	0.2	0.25	0.33	1	1	1	2	3	4	0.171	0.315	0.564
A4	0.25	0.3	0.5	0.25	0.3	0.5	0.25	0.3	0.5	1	1	1	0.057	0.082	0.151
A1	1	1	1	0.33	0.5	1	0.25	0.3	0.5	0.3	0.5	1	0.061	0.098	0.210
A2	1	2	3	1	1	1	1	2	3	0.2	0.25	0.33	0.102	0.225	0.440
A3	2	3	4	0.33	0.5	1	1	1	1	0.2	0.25	0.33	0.113	0.203	0.381
A4	1	2	3	3	4	5	3	4	5	1	1	1	0.256	0.472	0.841
K4															
A1	1	1	1	1	2	3	0.25	0.3	0.5	1	2	3	0.114	0.267	0.581
A2	0.33	0.5	1	1	1	1	1	2	3	1	2	3	0.116	0.277	0.620
A3	2	3	4	0.33	0.5	1	1	1	1	0.3	0.5	1	0.128	0.252	0.542
A4	0.33	0.5	1	0.33	0.5	1	1	2	3	1	1	1	0.093	0.202	0.465

Table 3

Ranking and selection of optimal technology

Fuzzy number				Weight priority vector	Final ranking
	L	S	D		
A1	0.008	0.055	0.366	0.176	3
A2	0.016	0.113	0.706	0.342	1
A3	0.019	0.113	0.664	0.325	2
A4	0.009	0.051	0.320	0.155	4

flects the "best" score. Hence, Alternative 2 (sublevel caving) is proposed as the best option or optimal mining technology in the given case. The second best is Alternative 3, followed by Alternative 4.

The proposed optimization procedure highlights the use of knowledge, intuition, and experience of engineers to gather all the necessary information about the natural system of a mine. Such an approach prioritizes mining safety and provides an efficient solution in terms of technical and economic conditions. Finally, and very importantly, the proposed solution needs to be implemented effectively. The ore excavation process and other mining activities would thus be standardized. This results in efficient mining as well as higher productivity and market competitiveness.

Conclusion. A modern approach to investigations in the field of geoscience and mine management was presented in the paper. The selection and design of a mining technology is one of the most complex tasks in mining engineering. Decision making requires the best possible knowledge about all the influencing parameters of the ore deposit. In addition to natural factors, decisions about the mining technology to be applied depend on technoeconomic aspects, ensuring of a safe and healthy work environment, and the possibility of sustainable ore extraction. An inadequate technology and its parameters can considerably reduce the benefits of mining, as well as bear upon economic viability and safety, which is one of the important criteria for decision making.

A fuzzy optimization approach is proposed due to the natural complexity of ore deposit structures, given that a large number of elements of the decision-making model are often uncertain and, in most cases, it is not possible to determine exact numerical values for comparing decisions. The proposed method makes use of a logical approach of the expert to create a problem hierarchy (objective $>$ criteria $>$ alternatives). A special application, Fuzzy-GWCS, has been developed to facilitate decision making with a fuzzy optimization model. Scores are entered and pairs compared in the application, and otherwise extensive mathematical calculations are simplified. It is also easy to monitor the sensitivity of the model to input parameter variation. On a much higher level, this heuristic approach copes better with mining management problems and offers a sustainable solution to decision makers.

It should be noted that this research opens the door for additional investigations, say the addition of other criteria like capital expenditure, physical and mechanical rock parameters, or environmental concerns. Likewise, the proposed methodology can be applied to consider other mining technologies when selecting the optimal alternative.

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REFERENCES

[1] Zadeh L. A. (1965) Fuzzy sets, Information and Control, **8**(3), 338–353.

[2] Zadeh L. A. (1975) The concept of a linguistic variable and its application to approximate reasoning, Information sciences, **8**, 199–249.

[3] Chang D. Y. (1996) Applications of the extent analysis method on fuzzy AHP, Eur. J. Oper. Res., **95**, 649–655.

[4] Zhu K., Y. Jing, D. Chang (1999) A discussion on extent analysis method and applications of fuzzy AHP, Eur. J. Oper. Res., **116**, 450–456.

[5] Tolga E., M. Demircan, C. Kahraman (2005) Operating system selection using fuzzy replacement analysis and analytic hierarchy process, Int. J. Prod. Econ., **97**, 89–117.

[6] Chen H. (2006) Applications of Fuzzy Logic in Data Mining Process. In: Bai Y., Zhuang H., Wang D. (eds), Advanced Fuzzy Logic Technologies in Industrial Applications, Advances in Industrial Control, London, Springer, DOI: 10.1007/978-1-84628-469-4_17.

[7] Bajić S., D. Bajić, B. Gluščević, V. Ristić Vakanjac (2020) Application of Fuzzy Analytic Hierarchy Process to Underground Mining Method Selection, Symmetry, **12**(2), 192, DOI: `https://doi.org/10.3390/sym12020192`.

[8] Namin F. S., A. Ghadi, F. Saki (2022) A literature review of Multi Criteria Decision-Making (MCDM) towards mining method selection (MMS), Resources Policy, **77**, 102676, DOI: 10.1016/j.resourpol.2022.102676.

[9] Čičić S., I. Avdagić, E. Čičić (2003) Geological map of Bosnia and Herzegovina, 1:300.000, Faculty of Civil Engineering, Sarajevo.

[10] Bajić D., D. Polomčić, J. Ratković (2017) Multi-criteria decision analysis for the purposes of groundwater control system design, Water Resources Management, **31**(15), 4759—4784, DOI: 10.1007/s11269-017-1777-4, 2017.

[11] Saaty T. L. (1990) How to make a decision: The analytic hierarchy process, Eur. J. Oper. Res., **48**(1), 9–26.

[12] Bajić D., D. Polomčić (2014) Fuzzy optimization in hydrodynamic analysis of groundwater control systems: Case study of the pumping station "Bezdan 1", Serbia, Geološki anali Balkanskoga poluostrva, **75**, 103—110, DOI: 10.2298/GABP1475103B ISSN: 0350-0608.

[13] Kwang H. C., H. J. Lee (1999) A method for ranking fuzzy numbers and its application to decision making, IEEE Transaction on Fuzzy Systems, **7**(6), 677–685.

[14] Lamata M. T. (2004) Ranking of alternatives with ordered weighted averaging operators, Int. J. Intell. Syst., **19**, 473–482, DOI: `https://doi.org/10.1002/int.20002`.

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